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MULTI-PRODUCT ALLOCATION AND DISTRIBUTION

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ABSTRACT

This paper describes techniques used by a beer company to optimize the allocation and distribution of their beer. These techniques are also applicable to other manufacturing and distribution environments.

Of particular interest is a two-phase model that allows a "look-ahead" to future sales, the decomposition of the problem into a large number of sub-problems (to reduce solution time), and the equitable distribution of material when stocks are over- or undersupplied.

The optimization is part of an interactive planning system, and the relationship between the optimization module and the rest of the system is briefly described.

1. INTRODUCTION

For operating purposes the Beer Division of the South African Breweries Ltd (SAB) has divided South Africa into three regions, namely Coastal, Northern Transvaal/Orange Free State and Southern Transvaal, of which the first two are further divided into independently managed sub-regions. For production, logistical and planning purposes the Southern Transvaal and Northern Transvaal/Orange Free State (including Northern Cape) regions have been grouped together into the Northern Provinces (NP); and the planning system described here is used by this region. The Systems Department of SAB is developing a similar system, tailored to the needs of the Coastal region.

In the NP region there are fifteen supply depots and six breweries; most of which have a depot on the brewery site. The beer is brewed and packaged at the breweries before being distributed to the depots, where it is sold to retail outlets.

SAB markets seven different brands of beer which are sold in eight types of packs (or containers). There are over thirty brand/pack (or product) combinations, but not all brands are packaged in the whole pack range, and only one brewery produces the full brand/pack range.

2. THE PLANNING PROBLEM

The NP and Coastal regions are each responsible for the beer requirements in their respective areas. This entails the development of quarterly and annual production plans, based on sales forecasts, for the weekly brewing, packaging and distribution of beer. Such a production plan has to demonstrate that there is sufficient brewing, packaging and distribution capacity to meet the forecasted sales, or, if this is not the case, it has to identify the causes so that timeous action can be taken. It must also facilitate the study of the impact of policy decisions, for example, regarding the utilization of breweries, on overall system performance and cost. Furthermore, it has to provide estimates of brewing, packaging and distribution volumes required for financial planning, the scheduling of labour and the ordering of raw materials.

Capacity limitations often only apply in particular instances. For example, there may be an insufficient capacity for brewing a particular brand at one brewery, even though there is sufficient brewing capacity at the other breweries. Similarly, there may be insufficient stocks of a brand/pack in one part of the region, while total stocks in the region are adequate. These problems can only be identified by examining, in detail, the consequences of a plan.

Distribution planning, in particular, is extremely complex and time consuming as there are over thirty different products that have to be distributed from the breweries to the depots. When evaluating annual plans, the planners have to calculate the impact of their decisions on all depot stock levels for each of fifty-two weeks.

Clearly, the brewing, packaging and distribution plans cannot be developed in isolation - brand brewing must meet the requirements for packaging of products, which in turn must meet inter-depot movements.

Budget planning is done at a weekly level instead of a monthly level for three reasons. First, in the shorter term it is essential that planning be performed on a weekly level. Second, the scheduling of brewing and packaging shifts is complex, and it is more convenient for the planners to think in terms of weeks, even when dealing with longer term plans. Finally, the finance department requires planning figures at a weekly level.

In the past the production and distribution plans were produced manually by a few experienced planners. This environment proved to be too complex for detailed planning. Furthermore, the manual process was laborious, and it was impossible to produce more than one plan within a certain time frame. Alternative scenarios could thus not be considered.

3. PRODUCTION PLANNING SYSTEM

PPS (Production Planning System), as described in Currin and Ittmann [1], was developed to facilitate the planning process. The main aim was to automate the planning process in order to alleviate the tedious parts of the job and to allow the planners to concentrate more on the planning aspects. By using this system, it is possible to study a proposed plan in the finest detail. The system includes modules to assist in the development of brewing, packaging and distribution plans. The driving concept of the system is that of interactive feedback, and the modules allow the planners to alter their plans and to evaluate the probable consequences of such changes. Indicators such as stock levels are calculated and shown on the computer terminal. The planners can then judge the suitability of these indicators, taking cognizance of any relevant external factors.

PPS uses both descriptive and normative techniques to aid planning. The descriptive approach does not prescribe solutions, but it does facilitate the planning process by relieving the planners of having to make tedious calculations. The major disadvantage of a descriptive approach, however, is that the planner may be unable to envisage the ramifications of his decisions when the problem is big. In such a case it is better to use a normative method to present the planners with a suggested plan. This plan can either be adjusted and then accepted, or the control parameters can be changed by the planner and a new plan can be requested.

A heuristic optimization routine is used to generate brewing and distribution plans for which the cost is at a minimum, subject to constraints imposed by the plans developed in the packaging module. The inter-depot (or distribution) plan can then be adjusted in the distribution module. The routine also allocates brands to breweries; this is used as a starting point for producing a brand brewing program.

The current paper does not discuss the descriptive components; instead, it focuses on the techniques used in formulating and solving the optimization problem.

Van Numen and Benders [2] describe a mixed-integer based system to assist in strategic and tactical decision-making at breweries. This system models the allocation of products to production lines, their subsequent distribution to warehouses, and their allocation to buyers. The system differs from PPS in that it concentrates on the distribution issues and considers brewing and packaging to be a single process. The model only deals with a single time period.

4. MODELLING CONSIDERATIONS

This section highlights some of the aspects considered during the design of the system.

Packaging is labour intensive, and, consequently, considerable attention must be given to the allocation of labour working in shifts on packaging lines, an area in which factors are not easily quantifiable. Management policy prescribes, among other things, the extent to which a brewery is utilized, which has a direct influence on the packaging plan. Therefore the optimization model subdivides the volumes of each pack to be produced among the various brands that can be produced at the breweries. The packaging planner can thus control the packaging plan fairly well, without being required to specify the plan in complete detail. This also implies that the allocation and distribution of brands can be treated as a separate problem for each pack. Consequently, a packaging plan for a particular pack can be changed and re-optimized, without the plans for the other packs being influenced.

The optimization model produces a brand/pack packaging plan for each of the breweries and subsequently determines where the brands should be brewed. There is still considerable latitude as to when brewing of the brands should be scheduled. As this scheduling has very little influence on costs, it is not included in the optimization component. Set up costs are minimal since changing from one brand to another simply involves washing out the brewing vessels and pipes.

Depot stocks are required as a buffer against fluctuations in sales and delays in inter-depot movements. These stocks are specified in terms of "days of sales", that is the number of days that the stocks would last if no new stocks were received and sales were equal to the forecasted values. As beer sales are highly seasonal, it is preferable to specify the desired stock levels in terms of "days of sales", rather than actual stock levels, since the latter can change drastically.

The brands that can be brewed at a brewery are determined by physical considerations as well as management policy. Similarly, not all inter-depot routes can be used. This means that even if there is sufficient brewing capacity in the region as a whole, it does not necessarily imply that demand at all the depots can be met.

5. MODEL FORMULATION

The model formulation is presented in two phases. In the first phase products that are not needed to meet depot requirements in a particular time period will not be produced until they are needed. The heuristic used in phase I waits until the products are needed before deciding which brands should be produced and to which depots they should be sent. The second phase revises the production and distribution plans to meet the prescribed packaging plan, and the brands are distributed to the depots before they are actually needed. This enables expected future sales to be taken into account when a distribution plan is determined. This is particularly important in planning how stocks should be increased prior to a period of peak demand. The precise method used is explained in the following subsections.

5.1 MODEL FORMULATION: PHASE I

In the first phase beer is only produced if it is needed to meet demand in the week that it is produced. The excess packaging capacity is "carried forward" and is used in subsequent weeks if necessary.

As the optimization of the allocation and distribution of the products can be treated as a separate problem for each pack, a pack index is not included explicitly in the sequel.

The following indices, variables and parameters will be used in the formulation of the phase I model.

Indices

i - brand indices;

k - brewery indices;

l - depot indices; and

s - time indices.

Decision Variables

x_{ikls} - quantity of brand i, distributed from brewery k to depot l in the specific pack during week s in the phase I formulation;

h_{ks} - excess packaging capacity at brewery k at the end of week s.

Parameters

c_{ik} - cost of brewing brand i at brewery k;

t_{kl} - cost of transporting the brands from brewery k to depot l;

p_{ks} - quantity of the specific pack packaged at brewery k in week s;

and

d_{ils} - demand for brand i at depot l in week s; this is calculated from the expected sales and the change in desired depot stocks.

The allocation and distribution problem can now be formulated as

$$\text{Minimize} \sum_{i,k,l,s} \{c_{ik} + t_{kl}\}.x_{ikls}, \tag{1}$$

subject to

$$\sum_{i,l} x_{ikls} + h_{ks} = p_{ks} + h_{ks-1} \quad \forall\ k,s \tag{2}$$

$$\sum_{k} x_{ikls} = d_{ils} \quad \forall\ i,l,s, \tag{3}$$

where $x_{ikls} \geq 0$, $x_{ikls} = 0$ if brand i cannot be brewed at brewery k or cannot be supplied to depot l.

The objective function is composed of brewing costs and transportation costs. Packaging costs depend on the volumes of each pack produced and not on the brands being packaged. Consequently, packaging costs are fixed and are not included. Holding costs are also omitted from the objective function since they are related to the desired stock levels.

The quantity $p_{ks} + h_{ks-1}$ on the right-hand side of (2) represents the available production capacity at brewery k in week s. It derives from two sources: the packaging capacity in week s, and the excess capacity of the previous period.

The right-hand side of (3) represents the amount of brand i demanded at depot l in week s. In general, this demand depends on three factors: the expected sales in week s, the desired opening depot stocks and the desired closing depot stocks. These desired stocks are calculated from the "days of sales" figures specified by the planners.

Notice that by summing the variables x_{ikls}, the required allocation of brands to breweries can be found. That is, $\sum_{l} x_{ikls}$ gives the amount of brand i to be packaged in the specific pack at brewery k in week s.

5.2 MODEL FORMULATION: PHASE II

The second phase adjusts the phase I solution so that all of the planned packaging capacity is used (the solution obtained from phase I might have excess packaging capacity). There is no optimization involved, but simply a shift in the time when the beer is packaged and distributed.

Define

y_{iklt} - quantity of brand i distributed in Phase II from brewery k to depot l in week t.

Calculate

$$\hat{x}_{ks} = \sum_{i,l} x_{ikls}$$

$$\hat{y}_{kt} = \sum_{i,l} y_{iklt} = p_{kt}.$$

The quantity $\hat{x}_{ks}$ represents the total quantity of beer of the specific pack distributed from brewery k in week s under the phase I solution. Similarly, the quantity $\hat{y}_{kt}$ corresponds to the solution that will be obtained from phase II, where packaged beer is distributed immediately (even if only to a warehouse at the brewing site). Consequently, it is required that $\hat{y}_{kt}$ equal the packaging plan p_{ks}, for s = t, specified by the packaging planner. The relationship between $\hat{x}_{ks}$ and $\hat{y}_{kt}$ is shown graphically in Figure 1.

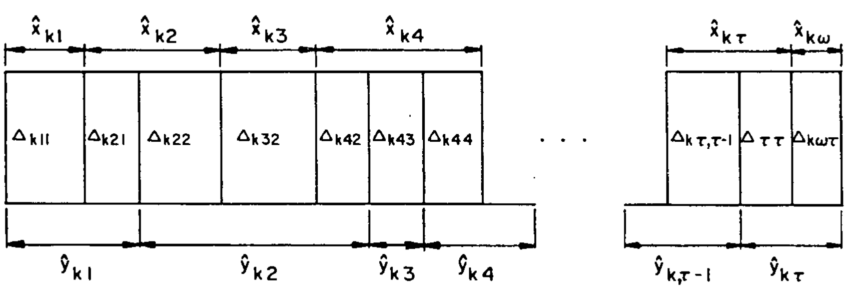

FIGURE 1

RELATIONSHIP BETWEEN SCHEDULES $\hat{x}_{ks}$ AND $\hat{y}_{kt}$

The quantities $\hat{y}_{kt}$ are equal to the quantity of packaging scheduled at brewery k in week t, whereas $\hat{x}_{ks}$ is the quantity planned according to the phase I distribution plan. The value Δ_{kst} represents the quantity scheduled for packaging at brewery k in week s of the phase I solution that is actually packaged at brewery k in week t of the phase II solution. The quantities Δ_{kst} that are not shown in Figure 1 are all zero. Notice that the cumulative sum of the $\hat{x}_{ks}$ variables until a given week is always less than or equal to the cumulative sum of the $\hat{y}_{kt}$ variables until the same week. This is because in phase I beer is distributed only if it is needed, whereas in phase II beer is distributed before it is needed.

The value $\Delta_{k\omega T}$ refers to the material distributed, but not actually needed to meet sales, in the last week. There may, in fact, be a few weeks at the end of the time horizon in which stocks that will not be needed within the time horizon are distributed; there will thus be no phase I solutions $x_{ikl\omega}$ corresponding to the variable $\hat{x}_{k\omega}$. The $x_{ikl\omega}$ variables used are taken to be an average of the x_{ikls} variables for the last four weeks.

The Phase II distribution plan will be made to accord with the packaging plan by adjusting it according to the values Δ_{kst}. Note that

$$\hat{y}_{kt} = \sum_{s} \Delta_{kst} = \sum_{s} (\Delta_{kst}/\hat{x}_{ks}) \cdot \hat{x}_{ks}.$$

Using this, we split $\hat{y}_{kt}$ into the distribution plan y_{iklt} in the same proportions that $\hat{x}_{ks}$ is split into the distribution plan x_{ikls}:

$$y_{iklt} = \sum_{s} (\Delta_{kst}/\hat{x}_{ks}) \cdot x_{ikls}.$$

Note that this is by no means the only way in which the split could be performed, but in this way all brands and depots are treated equally.

6. SOLUTION: PHASE I

The problem is a linear program and, in principle, it could be solved to obtain an optimal allocation and distribution plan. In practice, however, the problem is too big for a solution to be obtained in this manner. In an environment with seven brands, six breweries, eight packs, fifteen depots and fifty-two weeks there are over 50,000 non-zero decision variables. Furthermore, several solutions need to be generated. A solution is used to evaluate policy decisions, for example, regarding utilization of breweries, and the problem has to be re-solved each time that the packaging plan is changed. There is thus a need for the problem to be solved rapidly.

It is well known that the solution of problems can be speeded up by decomposing the problems into a number of separate problems. The problem, as presented, is already decomposed with regard to packs. Since there are 52 weeks, tremendous computational savings can be achieved by splitting the problem up over weeks. The desired depot stock levels are used to mediate the decomposition of the problem, by a demand at the depots being specified for each week. Notice that for a particular week the problem is in the form of a transportation problem, with supplies of $p_{ks} + h_{ks-1}$ at each brewery k and demands of d_{ils} for brand i at each depot l. The excess capacity h_{ks} can be regarded as going to a dummy demand point.

The problems for each week are solved sequentially. First, one will have to solve the problem for week 1, trying to achieve the stocks specified by the inter-depot planner and trying to minimize costs. The procedure will have to be repeated for subsequent weeks, with the stocks available at the end of the preceeding week being used as opening stocks. These sub-problems can be solved by using a variant of the transportation algorithm, which attempts to meet the desired stock levels, and if it cannot do so, equalizes the under- or oversupplies to the different depots. Cost minimization is used as a secondary objective. To meet desired stock levels is taken as being the first priority since failing to do so can lead to depots running out of stock. Details can be found in Currin [3].

7. Phase II Solution

The phase II solution is based on finding the relationship between schedules $\hat{x}_{ks}$ and $\hat{y}_{kt}$, in other words calculating the values Δ_{st}. These can be obtained by using a "north-west corner" type of rule. Suppose there are τ weeks.

(a) Set $t = 1$, $s = 1$;

(b) let $\Delta_{st} = \min\{\hat{x}_{ks}, \hat{y}_{kt}\}$;

(c) set $\hat{x}_{ks} = \hat{x}_{ks} - \Delta_{st}$, and $\hat{y}_{kt} = \hat{y}_{kt} - \Delta_{st}$;

(d) if $\hat{x}_{ks} = 0$, set $s = s+1$, otherwise set $t = t+1$;

(e) stop when $t > \tau$ or $s > \tau$, otherwise return to (b).

This method assigns material $\hat{x}_{ks}$ from the first weeks to be packaged to the first weeks when there is packaging capacity $\hat{y}_{kt}$. Material from later weeks is packaged later.

8. EXTENSIONS

In practice, the brands that can be brewed at a brewery and the inter-depot routes that are permitted can change during the planning horizon. When phase I is solved it is not known when the beer will actually be packaged and, consequently, whether a brand or a route will be permissible. This problem is resolved by introducing a new "brewery configuration" for each change in permissible brands or routes. Associated with each brewery configuration are its permissible brand/route combinations. The packaging volumes for any given week are assigned to the brewery configuration that applies during that week. Thus, in phase II, when the plans are altered to meet packaging, we can be sure that only the valid brand/route combinations will be used.

Previously, it was stated that the optimization problem can be solved independently for the various packs. If, however, the storage capacity of depots is a limiting factor, there may be an interaction between the different packs - the more stocks there are of one pack, the less room there is for other pack stocks. This constraint is not included in the optimization model as it is assumed that the desired depot stocks do not exceed the storage capacities of the depot. If this assumption does not hold, it indicates either that the desired stocks are unrealistic, or that the depots should be expanded.

The module for inter-depot planning allows the planner to alter the distribution plan in order to alleviate congestion at depots. It enables the planner to expedite or defer the transfer of beer between depots. Because costs are assumed to be independent of time, this cannot alter the optimality of the plan. It does, however, give the planner the facility to adjust the depot stock levels. For example, by deferring transfers of beer to a congested depot the stocks at that depot can be reduced. This technique can also be used to smooth weekly fluctuations in movements between depots.

The planner specifies the adjustments in aggregate, and they are then automatically split into adjustments for each product. While some products may be in plentiful supply, others may be in short supply. The adjustments are only made to products that will not drop below their minimum required level.

9. IMPLEMENTATION

PPS was developed in APL, using a fast cycle development approach, as this language is exceptionally suitable for rapid development.

The optimization routines were also written in APL so that they could interface easily with the other modules of PPS. This has the disadvantage, however, that because APL is an interpreted language, it takes longer to execute than a compiled language would. APL performs best when operating uniformly on large arrays since the overheads of interpreting are then minimal. For algorithms of the transportation type the labelling routines (Ford and Fulkerson [4]) have to work with scalars, and the overheads become significant.

The optimization of the annual budget takes approximately two hours of real time to solve, and it uses about twenty minutes of CPU time on an IBM 3083 computer, while the planners will spend from a few hours to a couple of days inputting and adjusting the plans. The optimization contributes significantly to the planning department's computing expenditure. This is, in fact, the major constraint to even more extensive use of the system. The authors are currently looking at ways of improving the efficiency of the optimization, as well as of all the other modules in the system.

The complete system has now been in use since January 1984. Initially, it was envisaged that the system would be used every three months to produce updated plans for the remainder of the year. In practice, it turns out that the system is used roughly once a month, with the packaging system being used even more frequently.

Quantifying the value of such a system is extremely difficult, but, as discussed in Keen [5], a number of benefits can be achieved. Using the system reduces the time required to produce a plan and gives the planners time to consider alternative scenarios. The total time required is not reduced, but the end result is a better quality plan. Because a plan is considered in greater depth, potential problems can be identified which might have been overlooked if the plan had been drawn up manually. The system enables less experienced planners to perform their jobs substantially better than they would have been able to do on their own. This means that the planning department is not so critically dependent on the availability of experienced planners.

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