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A REVIEW OF SIMPLE MULTIPLE CRITERIA DECISION ANALYTIC PROCEDURES WHICH ARE IMPLEMENTABLE ON SPREADSHEET PACKAGES

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ABSTRACT

A number of modern multi-criteria decision making aids for the discrete choice problem, are reviewed, with particular emphasis on those which can be implemented on standard commercial spreadsheet packages. Three broad classes of procedures are discussed, namely the analytic hierarchy process, reference point methods, and outranking methods. The broad princples are summarized in a consistent framework, and indications are given as to the practical aspects of implementing these on a spreadsheet. LOTUS spreadsheets implementing these are available from the author.

1. INTRODUCTION

Multiple Criteria Decision Making (MCDM) is fast becoming a major branch of operations research. For example, the 1988 EURO/TIMS international conference had 1½ streams (out of 30 streams, each consisting of 10 sessions) entirely devoted to the topic (the other ½ stream being the related area of group decision support), and a further two sessions on multi-objective optimization, while MCDM models received mention in many other streams. (By contrast, linear programming was allocated only 7 sessions in all.) The circulation of the Newsletter of the International Society on MCDM is close to 1000. Applications of the new MCDM methodology, in both private and public sectors, are also regularly reported; a few examples from 1988 are the following: industrial capacity expansion (Reeves, et al., 1988), portfolio analysis (Martel et al., 1988), manpower planning (Silverman et al., 1988), national energy planning (Capros et al., 1988) and environmental problems (Ellis, 1988), as well as two with a more South African flavour: fisheries management (Stewart and Brent, 1988; Stewart, 1988) and wildlife management (Jordie and Peddie, 1988).

This interest in the MCDM problem arises from the rapidly growing realization that human preferences are complex, and not reducible to simplistic objective functions such as cost minimization or nett profit maximization in any but the lowest level operational problems. In fact, it is even debatable whether terms such as "cost" or "nett profit" are themselves unambiguously definable in most realistic problem settings. If OR is to make contributions to strategic planning and decision making, we have to come to grips with the MCDM problem.

A wealth of approaches to the MCDM problem are available in the literature, but to the non-specialist, the plethora of approaches is more a source of confusion than help, in spite of the availability of recent reviews of the subject (eg. Goicoechea et al., 1982, and Steuer, 1986, although the latter is written in the context of multiple objective linear programming). Even when the techniques themselves are mastered, there is all too often no easily available software for implementing these techniques. Good quality commercial software incorporating MCDM concepts is only available for a few specific techniques or applications, and is in many cases either expensive or not well-supported. Yet, many of the approaches to MCDM which have been most successful are relatively simple in basic concept, and quite easily implemented. In this paper we review some of these simple approaches, particularly in the context of implementation on a spreadsheet package. (The macros reported have all been implemented in LOTUS Version 2. - see postscript at the end of this paper.) These spreadsheet implementations are really only suitable for relatively small problems, of (say) up to 20 or 30 alternatives and 6 or 8 decision criteria; nevertheless, many important problems are of this size, while working on problems of this size can assist greatly in developing intuitive understanding of the underlying MCDM principles.

In Section 2 we formulate and summarize the basic structure of the multiple criteria decision making problem, and outline some of the pitfalls of adopting naive methods of analysis. Three broad and quite distinct approaches are surveyed in Sections 3-5, namely the Analytic Hierarchy Process, Reference point methods, and Outranking approaches. Each have their place for certain classes of decision problem, and an indication is given in each case as to how each approach can be implemented in a LOTUS spreadsheet. Finally in Section 6, some other approaches and new trends are mentioned briefly for completion, although these do notappear suited to a spreadsheet implementation.

2. THE MCDM PROBLEM

Let us begin then with a formal definition of the multiple criteria decision making problem. We suppose that the decision maker (DM) is required to choose one <u>alternative</u> from a finite set of available options, labelled $\{1,2,\ldots,n\}$ say. These options may be physical objects (eg. different makes of computer), or may simply be plans of action (eg. routes for a new highway, or marketing strategies). Each alternative is assumed to be described by a set of <u>attributes</u>, labelled $\{1,2,\ldots,p\}$ say. These attributes are meant to describe meaningful measures of the extent to which each alternative contributes to the achievement of the overall objectives of the DM. Keeney and Raiffa (1976, chapter 2) discuss comprehensively the process of hierarchically decomposing a general goal (usually expressed in quite fuzzy terms such as "maximize welfare to society" or "maximize long-term profitability") into meaningful and measurable attributes. They recognize certain desirable properties to be satisfied by the set of attributes chosen, viz. the following:

<u>COMPLETENESS:</u> All important areas of concern must be addressed by the set of attributes.

<u>OPERATIONAL:</u> The attributes, and the scales in which they are

measured must be meaningful to the decision maker; the attributes should also be expressed in relatively neutral terms, as opposed to emotive terms, or terms which may be potentially embarassing politically.

<u>DECOMPOSABLE:</u> As far as possible, one should be able to discuss the desirability of potential trade-offs between any two attributes, assuming that the other attributes have constant values, without consideration of what these constant values are. (For example, we would wish when comparing possible cars to purchase, to be able to discuss allowable trade-offs between fuel economy and space, "all other things being equal", without having to ask whether all cars under consideration are red or green.)

<u>NONREDUNDANCY:</u> Take care not to count the same objective twice.

<u>MINIMUM SIZE:</u> Try not to let the number of attributes explode out of bound, in aiming at "completeness"; recall that the human mind does not easily cope with more than about seven stimuli at a time.

The completeness property would imply that the set of attributes is sufficient to define each alternative fully for purposes of decision making. Thus in choosing between new cars, we may for example accept that, for purposes of decision making, each car can be defined purely in terms of measures of price, fuel efficiency, availability of maintenance, spaciousness, top speed and comfort; even though two models may differ on many other aspects (eg. colour range, status), we may hold that these other aspects should not "rationally" affect our choice, and should thus not be part of the attribute set. (But the attribute set must be agreed by the DM!) In this view, an alternative j will be represented by a vector $\underline{z}^j$ of attribute measures $(z_1^j, z_2^j, \ldots, z_p^j)$, where z_i^j is the value of alternative j in terms of attribute i. For purposes of describing the principles of each MCDM method, and without loss of generality, we shall suppose that each attribute measure is defined in such a way that larger values are preferred to smaller. (For example, if cost is an attribute, we might define the negative of cost as our measure.)

The matrix of z_i^j values is of course easily maintained in a spreadsheet file. This is illustrated in Figure 1, which gives the data for a simple decision problem (which is the problem of selecting between ten cars, taken from the paper by Jacquet-Lagreze and Shakun, 1984, and which we shall use as an example to illustrate a number of the approaches). Note that for purposes of entering and editing data, we are not restricted to defining all attributes in an increasing sense. At the head of each column (representing an attribute) is a keyword "min" or "max", to indicate whether minimization or maximization of the attribute values is desired. Elsewhere in the spreadsheet, the input values can have the direction of preference reversed by use of the LOTUS @IF(.) function: for example, the price measure in cell E11 could be referenced elsewhere in the spreadsheet by @IF(E$1=$M$1,-E11,+E11), where cell M1 contains the string variable "min".

If for two alternatives j and k, $z_i^j \geq z_i^k$ for all attributes i, with strict inequality for at least one attribute, we need give no further consideration to alternative k: we say that j dominates k. If one alternative dominates all others, then there is no real decision to be made. In most cases, however, there will be many non-dominated (also called efficient, or Pareto optimal) alternatives from which a choice has to be made.

In theory, a set of quite mild axioms are sufficient to demonstrate the existence of a utility function $U(\underline{z})$ defined on the attributes, such that alternative j is better than alternative k (in terms of the DM's fundamental preference structure) if and only if $U(\underline{z}^j) > U(\underline{z}^k)$. In fact, by invoking an assumption termed "preferential independence" between the attributes (meaning roughly that there is an absolute value scale for each attribute independent of other attributes), Keeney and Raiffa (1976, Chapter 3) have shown that $U(\underline{z})$ has the separably additive form:

$$(1) \qquad U(\underline{z}) = \sum_{i=1}^{p} u_i(z_i)$$

where each $u_i(z_i)$ is a marginal utility function for its corresponding attribute. This result is deceptive, and often misused as a justification for grossly incorrect analyses done under a guise of objectivity and rationality. The marginal utilities have to be of a very specific form for the additive result to hold. There is no guarantee that the additive result will hold if the $u_i(z_i)$ are "approximated" by some weight multiplied by the measure z_i, even if the latter is some subjectively evaluated score. Apart from the psychometric problems of measuring weights at all, and the evident fact that the same weights can lead to very different conclusions if the scale of measurement of attributes is changed, there is also the mathematical fact that linear weighted sums of scores tend to generate more extreme solutions (very good on some and very bad on other attributes) than appears to be consistent with human preferences. This is illustrated by the hypothetical two-attribute problem depicted graphically in Figure 2: <u>any</u> linear weighted sum of scores will select either alternative A or alternative B, even though in many circumstances an alternative towards the middle of the sequence will be much more desirable (i.e. a better compromise between the conflicting goals). In two dimensions the problem is easily identified: not so in higher dimensions, where even substantial sensitivity analysis on the weights may continually yield the same extreme solution.

There are then really only two defensible paradigms for providing analytical support for MCDM problems. The first is to follow the Keeney-Raiffa prescription in full: but this is in general very time-consuming, and effort-intensive for the DM, which is not justified in many situations. (This effort may however be justified in large public sector decisions, where a clear record of the rationale behind the decision-making process may be needed for later public defence thereof.) The other paradigm is that of interactive decision support: no pretense is made of any "objective" "optimization", but a decision support system is provided, to guide the DM's judgement, and to focus it on the critical issues requiring human judgement. Partially assessed approximate utility functions may be used in the background, but only to eliminate clearly inferior alternatives and/or to suggest where the DM should look next. This is the direction taken by most recent MCDM research, and is the focus of this review as well. Our emphasis is on MCDM decision support tools which can easily be implemented on a spreadsheet package; not all methods are suitable for such implementation however, and in the final section we shall briefly mention other approaches.

3. ANALYTIC HIERARCHY PROCESS (AHP)

The Analytic Hierarchy Process (AHP) was developed by Saaty (1980), and perhaps popularized by the availability commercially of a comprehensive package (under the name "Expert Choice") incorporating the methodology. It should be emphasized that the AHP is more than just a technique for analyzing multi-criteria decision problems as formulated Section 2: it is a process in which the emphasis is on hierarchically decomposing the overall decision goal into objectives, sub-objectives, sub-sub-objectives, until a desirable set of attributes is achieved. Furthermore, in AHP it is not necessary, as we shall see below, for the attribute values z_i^j to be measured or stated explicitly; it is sufficient to be able to give operational meaning to the concept of comparing two alternatives in terms of attribute i. For the purposes of our discussion here, however, we are assuming that the attribute structure has been fixed, in terms of measurable attributes, and we discuss the use of the AHP methodology in this restricted context. For ease of comparison with other methods, we shall limit discussion to the MCDM problem as formulated in Section 2, which in AHP terms represents a single hierarchical level for the objectives. There is however no reason why the spreadsheet implementation we shall discuss cannot be extended to the more general hierarchies discussed by Saaty (1980).

In describing the AHP, it is important to differentiate between the preference model used, and the method of estimation of the parameters of this model. A lot of confusion has arisen in some quarters by not recognizing this distinction. The preference model itself is essentially that given by (1), and is thus implicitly based on the same axiomatic foundation, although in AHP the equation is in effect expressed in the following form:

$$U(\underline{z}) = \sum_{i=1}^{p} w_i v_i(z_i) \tag{2}$$

where $v_i(z_i)$ is an intrinsic worth, in terms of attribute i, of an alternative having a measured value z_i on this attribute. It must again be emphasized that the z_i values are generally measured on convenient physical scales (eg. Rands; ppm pollutants; etc.), and not on preference scales; $v_i(z_i)$ <u>is</u> measured on a preference scale, and will in general not be a linear function of z_i. The parameters w_i are then weights representing the absolute contribution of each attribute to the overall goal. Implicit in (2) is the assumption that the preference value scales represented by the functions $v_i(.)$ are equivalently scaled, so as to allow comparisons between attributes; Saaty normalizes the attribute scales such that for the given set of alternatives:

$$\sum_{j=1}^{n} v_i(z_i^j) = 1$$

for each attribute i. This makes the scaling dependent on the relative distribution of values in the set of alternatives. Other approaches have sought to minimize this dependency by other scaling, for example forcing the minimum and maximum values to be 0 and 1 respectively. There is merit in both approaches, but fortunately in most practical examples it seems not to make too much difference which is used. See, however, Belton and Gear (1983,1985) and Saaty and Vargas (1984) for some discussion.

The user is asked to provide preference information in the form of comparisons, firstly between attributes as to their contribution to achievement of the overall goal, and secondly between alternatives as to their worth in terms of each attribute taken in turn. The comparisons are on a nine-point scale (1=equal importance; 3=weak preference for one over the other; 5=essential or strong preference; 7=demonstrated preference; 9=absolute preference). The fundamental assumption of AHP is that this scale of responses is a ratio scale of preferences, i.e. that if a response a_{jk} is given by the DM (when comparing either two attributes j and k in terms of contribution to overall goals, or two alternatives j and k in terms of some specified attribute), then to some level of approximation:

$$a_{jk} \approx w_j/w_k \qquad \text{for two attributes}$$

or

$$a_{jk} \approx v_i(z_i^j)/v_i(z_i^k) \qquad \text{for two alternatives w.r.t. attribute } i.$$

The justification for the ratio scale assumption is largely empirical.

Thus far we have discussed the preference model aspect of AHP. To use the model it is necessary to use the responses to estimate all the relevant values w_i and $v_i(z_i^j)$, for $i=1,\ldots,p$ and $j=1,\ldots,n$. Saaty proposes use of the principal eigenvector of the matrix of a_{ij} values for each set of comparisons. This is tedious and time consuming to attempt in a spreadsheet framework. It has however been noted that other estimation procedures can also be justified (cf. Cogger and Yu, 1985, and Cook and Kress, 1988), and for reasonably consistent sets of responses, the actual method of estimation is not very critical. For spreadsheet implementation, a least-squares fit is particularly useful: using the symbol π_j to represent either w_j, or $v_i(z_i^j)$ for some i, as the case may be, we estimate the set of values π_j by minimizing the following expression:

$$\sum_{j=1}^{m} \sum_{k=1}^{m} (a_{jk}\pi_k - \pi_j)^2 \tag{3}$$

subject to the constraint:

$$\sum_{j=1}^{m} \pi_j = 1 \tag{4}$$

where m=p or n as the case may be, and where a_{jj} is defined to be 1. It is easy to see by simple differentiation of the Lagrangian, that the solution is in principle obtained by solving the following set of linear equations:

$$(m + \sum_{k=1}^{m} a_{kj}^2 - 2)\pi_j - \sum_{\substack{k=1\\k\neq j}}^{m} (a_{jk} + a_{kj})\pi_k = \mu \tag{5}$$

$$j=1,2,\ldots,m$$

where the constant μ is chosen so that (4) is satisfied. In fact, it is easy to verify that (5) can be solved by initially using any arbitrary value for μ, and then simply rescaling the solution so as to satisfy (5).

One practical problem is that if the DM is fully consistent in every assessment (i.e. if $a_{kj}=a_{kr}a_{rj}$ for any k, j and r), then one of the equations in (5) is redundant, and the resulting matrix is not invertible. This can be circumvented by adding a small positive quantity (0,01 seems to suffice) to the coefficient of π_j in (5). This results in a slight shrinkage of the π_j values towards each other, but ensures a stable numerical solution in each case.

With this form of parameter estimation, the basic AHP is easily implemented on a LOTUS spreadsheet. Tables of comparisons can be set up anywhere in the spreadsheet, with 1's down the diagonal, for both the comparisons between attributes and those between alternatives. This is illustrated in Figure 3 for the problem of choosing between three schools, on the basis of six attributes, which is discussed by Saaty (1980); in Figure 3 is shown the comparison matrix for the attributes, and those for comparing the three schools in terms of two of these attributes. A macro is constructed, which sets up the linear equations (5), including the small positive increment to the coefficient of π_j, and using μ=1 say, for any comparison matrix, and solves these using the LOTUS /Data Matrix Invert and Multiply menu options. For comparison, the weights w_j obtained in this way, and those by Saaty's eigenvector approach, are as follows for the attributes in Figure 3:

	least squares macro	eigen= vector
learning	0,41	0,32
friends	0,09	0,14
school life	0,03	0,03
vocational train.	0,11	0,13
college prep.	0,22	0,24
music	0,13	0,14

In another part of the spreadsheet, an array containing the values of $w_i v_i(z_i^j)$ can be set up (expressed as formulae, of course), from which can be obtained a column of values for $U(\underline{z}^j)$ using @SUM(.). If desired these can be sorted to give a preference rank ordering of the alternatives. The beauty of the spreadsheet approach, is that the user can inspect and modify the entries in the comparison matrices at any time, and re-invoke the estimation macro, after which the values for $U(\underline{z}^j)$ for each alternative j, will also automatically be updated in the usual manner.

4. REFERENCE POINTS AND SCALARIZING FUNCTIONS

In essence, the AHP scores over full estimation of $U(\underline{z})$ (per Keeney and Raiffa), and thus qualifies as an interactive MCDM aid, because it estimates values of $u_i(z_i)$ only at the finite number of points at which it matters, and not over the whole space of possible outcomes. This eliminates a lot of the unnecessary cognitive demands on the decision maker; AHP is nevertheless still a method of establishing rigorously a preference value function, and quite quickly becomes a time consuming and tedious process as the size of the problem increases. From practical considerations, therefore, its application is limited to relatively small numbers of alternatives (perhaps up to 7 or 8), which have very rigorously and objectively to be compared. In this Section and the next, we introduce other interactive methods which do not explicitly estimate a value function, and which in fact also require somewhat weaker assumptions than those needed to demonstrate the existence of the additive value function (1).

In this Section we discuss methods which are based broadly on the idea of "goal programming", and which have the advantages of being applicable to large numbers of alternatives (the constraint being ultimately computational, and not cognitive), and of being very simple in concept to allow explanation of the workings to the DM (following the precept in Woolsey and Swanson, 1975, p71: "A manager would rather live with a problem he cannot solve than accept an answer he cannot understand").

Following the convention which we introduced in Section 2, that larger values of z_i are preferred to smaller, let us define the following concepts:

$$\underline{\text{Ideal value}}:\ I_i = \text{Maximum}_{(1\le j\le n)}\ z_i^j$$

$$\underline{\text{Nadir value}}:\ N_i = \text{Minimum}_{(1\le j\le n)}\ z_i^j$$

To these, let us add a third concept, namely that of a <u>Reference level</u>" R_i; this will generally be defined, and subsequently modified interactively, by the DM, and will in some sense be understood to mean a <u>realistically desirable value</u> for an attribute. The literature is not always clear on precisely what is meant by this term, and there are three possible meanings as follows:

(1) A level for the attribute which <u>must</u> be achieved for any alternative to be satisfactory;

(2) A level which the DM would <u>wish and expect</u> to achieve in the solution adopted (a "target" level);

(3) A level that is probably unattainable, but which the DM ideally would <u>desire</u> to be able achieve (a "satisficing" or "aspiration" level for the attribute).

In some cases the methods are not very sensitive to which of these three meanings apply, but in other cases it may be important to ensure that the DM does understand what it is that is being asked. We shall thus indicate the specific meanings in the discussion below.

The basic reference point method as introduced by Wierzbicki (1980) is not unduly sensitive the precise meaning of reference level, but is nevertheless best understood and implemented in the context of meaning (2) above: a target level which the DM can realistically aspire to. The essence of the idea is that of a "scalarizing function" $S(\underline{z};\underline{R})$, which measures the degree to which each alternative fails to satisfy the targets implied by the specified reference levels R_i. If the target levels R_i cannot all be achieved simultaneously, then minimization of the scalarizing function is meant to generate the alternative which is "closest" in some sense to these levels; otherwise, the closest Pareto optimal solution satisfying all target levels should be identified. Choice of the form of scalarizing function has been discussed by Wierzbicki (1980) and by Lewandowski and Grauer (1982), in fairly general mathematical terms. For many practical purposes (including spreadsheet implementation), the following form has been found to be particularly satisfactory:

$$(6) \qquad S(\underline{z};\underline{R}) = \operatorname*{Max}_{\{1\le i\le p\}} \beta_i[R_i - z_i] + \epsilon\sum_{i=1}^{p} \beta_i[R_i - z_i]$$

where $\epsilon>0$ is a suitably small positive quantity, and the β_i are scale factors to ensure comparability between attribute scales (which we shall discuss shortly).

The approach is then for any given reference point (vector $\underline{R}$), to select the alternative j which minimizes $S(\underline{z}^j;\underline{R})$. The "Max" term is clearly dominant ,and what this does in effect is firstly to reduce maximum relative deviation below the reference level, and then, if all reference levels are achieved, to increase the minimum over-achievement of these levels. The process is thus somewhat conservative, but ensures that no criterion is seriously disadvantaged, and is consistent with what appear to be the true goals of "satisficing" behaviour. It must be re-emphasized that the process is designed to be interactive: the DM sets initial reference levels R_i, and the alternative minimizing $S(\underline{z}^j;\underline{R})$ is identified as a first tentative solution. (In fact, if the DM feels unable to specify R_i values initially, these can be set at say $(I_i + N_i)/2$ to start.) By examining the attribute values in $\underline{z}^j$, the DM can modify these (by considering which need improvement, and on which sacrifices may be acceptable) to form a more realistic reference level. The process is repeated until the same $\underline{z}^j$ recurs continually.

A definition for the scale factors which has been suggested is:

(7) $$\beta_i = 1/[I_i - R_i].$$

This certainly makes the scalarizing function independent of the specific units of measurement of the attributes, and also weights attributes with higher target levels more than others (which makes sense: note that we specifically avoid asking the DM to specify importance "weights" which are intuitively difficult to interpret in this context). A problem does arise when $R_i - z_i < 0$ for all attributes: in this case the weights work the wrong way round in the "Max" term, because the maximum term becomes the least negative one, and the smallest weight becomes controlling. For realistic target levels this should not happen to any great extent, and the contingency is often ignored; an alternative is to define the scaling by:

(7)' $$\beta_i = 1/[I_i - N_i].$$

The basic reference point approach is so easily implemented on a spreadsheet that it hardly requires explanation. Apart from the input data matrix, provision has only to be made for the DM to enter reference levels for each attribute (which can initially be set to $(I_i-N_i)/2$ say). The ideal and nadir values are generated using @MAX(.) and @MIN(.) functions. A further array is set up containing the formulae for $(R_i - z_i^j)/(I_i - R_i)$ with the alternatives as rows and the attributes as columns (for consistency with Figure 1); a further column can be used to set up scalarizing function values (6) for each alternative j. At any stage, the DM can use /Data Sort to order the alternatives according to (6).

A practical problem which has been experienced arises from the difficulty in specifying a realistic reference level initially, coupled with the natural human resistance to giving up gains. DMs tend to be very conservative in stating how much they are prepared to give up on other attributes in order to improve the less satisfactory attributes (in the current solution). The result is that the process terminates too early (often within one or two iterations), because no room is allowed for compromise to be discovered. One way to overcome this problem is (a) to urge DMs to start with quite low target levels (perhaps by forcing N_i as the initial value), and to increase these slowly, even one at a time; and (b) to provide as additional information the so-called potentials defined by:

$$P_i = \text{Maximum}_{\{j \mid z_k^j \geq R_k \text{ for all } k\}} z_i^j$$

i.e. the best that can still be achieved on each attribute, if the target levels are set as hard constraints. The potentials can easily be set up as a row in the spreadsheet, and then give an immediate indication to the DM of the damage done to other criteria by being too demanding on one. It is also useful in the spreadsheet to create a flag (using the @IF(.) function) indicating which alternatives would be eliminated if the targets were enforced as constraints.

This modified procedure results in slower convergence to a solution, but some experiments have suggested that the results are closer to true preferences (cf. Stewart, 1988). The gradual increase in reference level from the bottom, coupled with information on potentials, was initially proposed in the context of a method termed Interactive Multiple Goal Programming, and developed by Spronk (1981).

5. OUTRANKING APPROACHES

While the "American School" of MCDM (if such a thing really exists) tends to favour the use (implicitly if not explicitly) of preference models of the form exemplified by (1), the "European School" has tended to favour less restrictive assumptions about human decision-making behaviour. A case in point is the outranking philosophy developed by Roy and co-workers in Paris, and implemented in the various versions of the ELECTRE method (see for example Roy, 1977; and Roy and Vincke, 1981). The concept of outranking has been defined in many ways, but a useful operational definition is to say that alternative j outranks alternative k if:

(i) the weight of criteria are in accord with the assertion that alternative j is at least as good as k (a "concordance" condition); and

(ii) there is no criterion which is in strong discordance with this assertion (a "discordance" condition).

The above conditions can be linked to the attribute values in an algorithmic manner. Define first the following indices:

$$\text{Concordance: } C_{jk} = \Sigma_{\{i: z_i^j \geq z_i^k\}} w_i$$

for some set of weights w_i representing the relative importance of each attribute (which have however much less extreme impact than have the weights in additive score models), and

$$\text{Discordance: } D_{jk} = \text{Max}_{\{1 \leq i \leq p\}} \left[\frac{z_i^k - z_i^j}{I_i - N_i} \right].$$

Alternative j may then be said to outrank k if:

(8) $$C_{jk} \geq c$$

and

(9) $$D_{jk} \leq d$$

for some pair of values c and d. If c is too large and/or d is too small, then no alternative outranks any other: all are seen to be good in their own dimensions, but are not comparable in any way. This gives no assistance in recognizing the better alternatives. Conversely, if c is too small and d is too large, then every alternative outranks every other: no evidence is then available to declare any alternative better than any other, which again gives no assistance for decision analysis. The trick is therefore to experiment with various values of c and/or d (a more risk averse DM may prefer to keep d quite small and vary only c, and vice-versa) until one, or a suitably small number of alternatives, outranks most or all of the others: this becomes the recommended decision, or a final short-list from which a final selection will be made by holistic judgemental evaluation.

It is perhaps tedious, but not fundamentally difficult to set up an outranking matrix in the spreadsheet. The most convenient form (for purposes of the partial ordering macro to be described below) is a square matrix, with alternatives represented by both columns and rows; a cell is defined to have a value of 1 if the "row" alternative (which we shall denote by j) is outranked by the "column" alternative (which we shall denote by k), and a value of 0 otherwise. Use has first to be made of the LOTUS /Range Transpose, to place the z_i^j values for a particular alternative j into columns as well as rows: this is best done as part of an initial macro invoked by the user to set up the interaction after completion of data entry. The matrix of concordance indices is set up using directly the formula defining C_{kj}. (Note how we have reversed the index order in order to emphasize that we shall be looking at the "column" alternative k outranking the "row" alternative j). It is less easy to set up a matrix of discordances directly; but in view of the way in which these are used in (9), it is sufficient to set up a matrix of terms M_{kj} say, defined to be the number of attributes for which:

$$(10) \qquad \left[\frac{z_i^j - z_i^k}{I_i - N_i} \right] > d.$$

This requires a sum of terms involving @IF(.) functions, giving 1 or 0 depending on whether or not the inequality (10) is satisfied. Then alternative k outranks alternative j if $C_{kj} > c$ and $M_{kj} = 0$; the outranking matrix we have defined becomes thus simply an array of @IF(.) functions. An illustration of such a matrix for the data in Figure 1, is displayed as Figure 4; here the relative weights on the four attributes are shown in the ratios 1:2:5:1, and the cut-off values for (8) and (9) are c=0,6 and d=0,4. (Ignore for the moment the "partial pref. ranking" column, which we discuss below.) The user can modify the weight ratios at will, and can experiment with the cut-off values as indicated above; the effect on the outranking matrix is seen immediately by the usual LOTUS Calculation key. (It is recommended in this situation that the worksheet be set to manual recalculation, so that the matrix is only updated after all changes have been made.)

The outranking information in the matrix can be interpreted without further computations; this is usefully done by setting up a graph of the outranking relations, i.e. a graph in which the nodes represent alternatives, and a directed arc from j to k indicates that j outranks k. For the outranking matrix in Figure 4, such a graph is depicted as Figure 5. Some evident patterns emerge. Firstly there is a separation into two sets of cars, with no cross-linking relationships. This implies that there is no basis on which to conclude from the preference information thus far, that either set is preferred to the other. The user must focus his judgement on what becomes the critical choice, viz. whether a small, cheaper car or a large, more expensive car, better matches his needs and objectives: the analysis will not do this for him! Then within each set, there is a unique winner (in fact a fairly well-defined ordering), which would become the recommendation depending on the small-cheap versus large-expensive choice made. Uniqueness does not necessarily always occur, and further key choices may have to be made to break deadlocks; the point however is that the DMs attention is always focussed on the critical judgemental choices, which is the fundamental aim of all MCDM analysis.

It is not however always convenient to draw out the outranking graph as in Figure 5. Various schemes have been suggested for turning the outranking matrix into a partial preference ordering (eg. Goicoechea et al., 1982, pp 197-203). A simple partial ordering that fits into the spreadsheet framework, and which works well if the cut-off parameters c and d do not allow too large a number of outranking relationships, is based on number of levels by which an alternative is outranked. Specifically, we define:

> <u>Rank 1</u> alternatives as those which are not outranked by any other alternatives;
>
> <u>Rank 2</u> alternatives are those which are not outranked by any except rank 1 alternatives;
>
> <u>Rank 3</u> alternatives are those which are not outranked by any except rank 1 and rank 2 alternatives;
>
> etc.

Although quite complicated looking, it is quite simple to write a LOTUS macro which:

- sets up a column of row sums of the outranking matrix, and turns these into zeros or ones depending on whether or not the entry is greater than 0 (i.e. a 1 denotes that the corresponding alternative is outranked at least once, and a 0 that the alternative has rank 1);

- multiplies the outranking matrix into this column (to give a row sum of the outranking matrix, but counting only columns corresponding to alternatives which did not have a zero in the previous column), places these into the adjacent column, and turns these into zeros or ones as previously (so that a 1 in this column denotes that the corresponding alternative is outranked at least once by alternatives other than rank 1 alternatives, and a 0 that the alternative has rank 2);

- and repeats this for as many columns as required to get no further change (or up to some fixed maximum number of times).

The rank of the alternative is easily seen to be the corresponding row sum of these columns plus one. The partial preference orders shown in Figure 4 were obtained in this way. Note the correspondence between these rank orders and the graph of Figure 5.

The main limitation of the outranking approach as implemented in this way, is that the number of alternatives cannot be too large in view of the fact that nxn matrices need to be manipulated. The approach does give many useful insights, however, particularly when the number of attributes is too large for comfortably using the other approaches.

FINAL COMMENTS: OTHER MODERN INTERACTIVE MCDM TOOLS

Not all MCDM approaches lend themselves to spreadsheet implementation. There are a number of methods which rely on some form of interactive on-line evaluation of the underlying utility function, based on pairwise comparisons or local trade-offs provided by the DM. The idea is that an alternative, or a pair of alternatives is shown to the decision maker, who is required to express some local preferences (choice between the two, or trade-offs between attributes that would be acceptable). This is used to constrain the range of utility functions which can apply, and on this basis certain alternatives may be elimininated and/or the DM can be guided to more promising alternatives. For details, see for example Korhonen, Wallenius and Zionts (1984) and Stewart (1989).

An important new technological development which is affecting all of OR, but specifically also the MCDM field, is the use of visual interactive graphics as an integral part of modelling. One fascinating MCDM implementation is the "Pareto Race" of Korhonen and Wallenius (1988): this is an approach to multiple objective linear programming, and is thus not directly applicable here, but is an attempt to provide a form of video-game allowing the user to drive around the set of Pareto optimal solutions, changing direction and speed at will. To some extent inspired by this, we (Lotfi, Stewart and Zionts. 1988) have attempted to design a scheme for the discrete choice problem, in which the user also explores the options systematically using the arrow keys on the keyboard. This is however still somewhat in an experimental stage.

These more sophisticated MCDM methods are not well-suited to spreadsheet implementation, because of the high computational overheads. Nevertheless, the spreadsheet format remains useful for preparing input data for these other methods; in fact a scheme can be envisaged whereby a spreadsheet data entry procedure for these other methods can be coupled with the methods suggested above, to be used as a pre-screening (to start the other method off in a good direction) or post analysis (of a short-list of options generated by the other method).

Postscript: All the spreadsheet procedures described above have been implemented by the author using LOTUS Version 2, for use on an IBM-compatible micro-computer. A diskette containing worksheets set up for each method, including all relevant macros, as well as a brief user documentation, is available on request from the author for a nominal handling fee of R50,00 (for orders received from within the Rand monetary area) or US$50,00 (elsewhere). Cheques should be made payable to the University of Cape Town Entity Number 3699.

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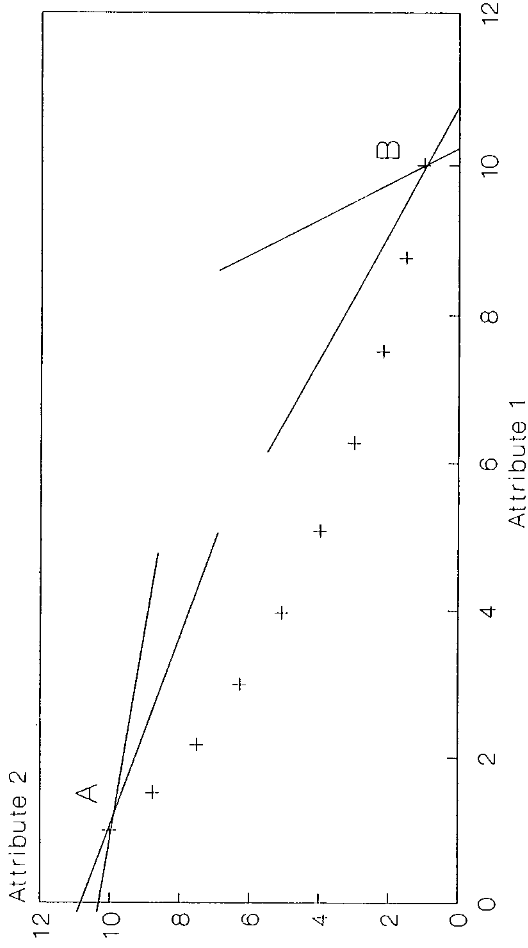
FIGURE 1: Illustration of MCDM problem formulation in a LOTUS spreadsheet

"min" or "max"	min	max	min	max
criterion	c120	space	price	speed
ideals:				
worst:				
reference point:				

	c120	space	price	speed
opel	10.48	7.96	46700	176
p505	10.01	7.88	49500	173
p104	8.42	5.11	35200	161
dyane	6.75	5.81	24800	117
visa	7.3	5.65	32100	142
golf	9.61	6.15	39150	148
m230	10.4	8.47	75700	180
cx	11.05	8.06	64700	178
volvo	12.95	8.38	55000	145
bmw	12.26	7.81	68593	182

FIGURE 2: Hypothetical two-attribute example



+ alternatives

(Solid lines represent different linear objective functions)

FIGURE 3: Illustration of some pairwise comparison matrices, and the corresponding estimated weight vectors, as implemented on a LOTUS spreadsheet.

learning	1	4	3	1	3	4	0.413408
friends	1/4	1	7	3	1/5	1	0.093865
school life	1/3	1/7	1	1/5	1/5	1/6	0.034941
vocational train.	1	1/3	5	1	1	1/3	0.112890
college prep.	1/3	5	5	1	1	3	0.219203
music	1/4	1	6	3	1/3	1	0.125691

learning:

1	1/3	1/2	0.174906
3	1	3	0.602829
2	1/3	1	0.222264

vocational training:

1	9	7	0.802391
1/9	1	1/5	0.074938
1/7	5	1	0.122670

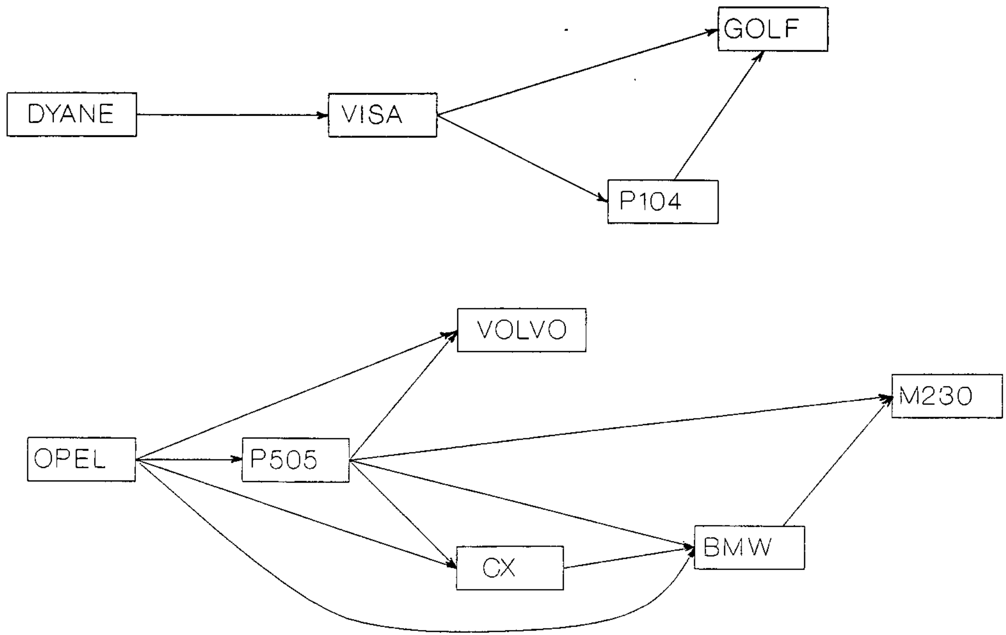
FIGURE 4: Illustration of an outranking matrix generated on a LOTUS spreadsheet

criteria:	c120	space	price	speed
weights:	1	2	5	1

critical values: concordance: 0.6 discordance: 0.4

	partial pref. ranking	OUTRANKING MATRIX: 1 IMPLIES COLUMN OUTRANKS ROW								
		opel	p505	p104	dyane	visa	golf	m230	cx	volvo	bm
dyane	1	0	0	0	0	0	0	0	0	0	0
opel	1	0	0	0	0	0	0	0	0	0	0
visa	2	0	0	0	1	0	0	0	0	0	0
p505	2	1	0	0	0	0	0	0	0	0	0
cx	3	1	1	0	0	0	0	0	0	0	0
p104	3	0	0	0	0	1	0	0	0	0	0
volvo	3	1	1	0	0	0	0	0	0	0	0
golf	4	0	0	1	0	1	0	0	0	0	0
bmw	4	1	1	0	0	0	0	0	1	0	0
m230	5	0	1	0	0	0	0	0	0	0	1

FIGURE 5: Graph depicting outranking relationships



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Letter to the Editor

Comments on "The Negotiation and Resolution of the Conflict in South Africa: the AHP". by Th. L. Saaty (ORiON), Vol. 4, No. 1, pp. 3–25, 1988).

It is well–known that Saaty has a deep interest in the solution of political conflicts and that he devoted much of his time and energy to solve them via his Analytic Hierarchy Process, a method in the field of multi–criteria analysis. The racial conflict in South Africa is probably one of the hardest problems in actual history. It prompted Saaty to develop a new extension of the AHP, using the estimated benefits and costs of possible concessions between two parties which are in conflict.

As an OR specialist, Saaty has a particular responsibility for the proposed methodology (this is always the case when OR specialists cooperate with users to solve managerial, administrative or political problems). Regrettably enough, we found several deficiencies in the methodology just mentioned. They should be thoroughly discussed before the OR community launches a professional contribution to the solution to the solution of the racial conflict.

Saaty tries to estimate the benefits and the costs of the concessions separately via the AHP. The white party, for instance, compares the black concessions in pairs under certain criteria; these criteria are also weighted via pairwise comparisons, and the results are finally aggregated into the final scores, the benefits of the black concessions as viewed by the white party. It is well–known, however, that the final scores have a multiplicative degree of freedom so that the benefits are not unique. The costs of the white concessions as viewed by the white party are calculated in a similar way. They also have a multiplicative degree of freedom. So, when all benefits and costs are evaluated, it is still impossible to estimate the cost/benefit ratios (the trade–offs) of the concessions offered by the respective parties. This is a glaring weakness of Saaty's proposal (see also ref. [1]). Moreover, Saaty does not motivate the subsequent operations on the estimated benefits and costs (multiplication of own benefits and adversary's costs of a concession, addition of these products in order to calculate the value of a possible deal between the two parties). In some cases, these values do not properly represent the feelings of the respective party members.

To illustrate the key issue, the multiplicative degree of freedom of the final scores in multi–criteria analysis, we observe that the criterion weights $c_i, i = 1,\dots,m$, and the impacts a_{ij}, $i=1,\dots,m$, $j=1,\dots,n$, of the alternatives

$A_1,...,A_j,...,A_n$, under the respective $C_1,...,C_i,...,C_m$, are not unique. They are usually normalized, in the sense that

$$\sum_{i=1}^{m} c_i = 1,$$

$$\sum_{j=1}^{n} a_{ij} = 1, \qquad i=1,...,m.$$

Nevertheless, the ratios a_{ij}/a_{ik} are unique. Hence, we can model the ratio s_j/s_k of the final scores corresponding to the pair of alternatives A_j and A_k by

$$\frac{s_j}{s_k} = \prod_{i=1}^{m} \left(\frac{a_{ij}}{a_{ik}}\right)^{c_i},$$

which yields

$$s_j = \prod_{i=1}^{m} a_{ij}^{c_i} \qquad \text{(geometric aggregation rule).}$$

Using first–order approximations we obtain the familiar, but approximate result

$$s_j \simeq \sum_{i=1}^{m} c_i a_{ij} \quad \text{(arithmetic aggreagtion rule).}$$

Obviously, the ratios s_j/s_k are unique, but the final scores have a multiplicative degree of freedom.

For Saaty's conflict analysis, these considerations have the following significance. We view the situation from the standpoint of part I, say. Let β_j denote the benefit of the j–th concessions offered by party II. The β_j, the final scores of a multi–criteria analysis by party I of all possible concessions of the adversary, are not unique; they have a multiplicative degree of freedom. Similarly, let γ_k stand for the cost of the k–th concession offered by party I itself. The γ_k, the final scores of *separate* multi–criteria analysis by party I of all their own possible concessions, do also have a multiplicative degree of freedom, but it is *not* the degree of freedom of the benefits β_j. Hence the trade–off β_j/γ_k of the deal where concession j is offered in exchange for concession k, has almost no meaning. We could equivalently write the trade–off as

$$\lambda\beta_j/\mu\gamma_k .$$

With arbitrary positive degrees of freedom λ and μ, but this ratio could be smaller or larger than 1 so that it remains unclear whether party I would accept or reject the deal in question. Multi–criteria analysis, as recommended by Saaty, fails to establish a "rate of exchange" between mutual concessions. The calculated trade–offs can only be used to rank all possible deals, but even then we cannot distinguish the deals which are in principle acceptable for party I.

It is not our intention to say that multi–criteria analysis is useless in a conflict. On the contrary, it enables each party to rank the benefits of the adversary's concessions as well as the costs of their own concessions. In doing so, the party members are urged to make up their minds about the critical issues in the actual conflict. Finally, we have no illusion about the power of conflict analysis. It is a common experience in multi–criteria analysis (mostly used in a situation with incompatible viewpoints) that human beings do not easily accept the results of a mathematical technique. Their behaviour is even worse in an open conflict. Finally, we have no illusion authe power of conflict analysis. It is a common experience in multi–criteria analysis (mostly used in a situation with incompatible viewpoints) that human beings do not easily accept the results of a mathematical technique. Their behaviour is even worse in an open conflict. Nevertheless, it will always be worth trying out the ideas, provided that the underlying methodology is sound.

June 1989.

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Reference

[1] F.A. Lootsma, Conflict via Pairwise comparison of Concessions. *European Journal of Operational Research* 40, 109–116, 1989.

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RESPONSE FROM: Thomas L Saaty, University of Pittsburgh

Slow down, Mr. Lootsma, and read carefully what the paper says before you launch into your objections.

What is important in the Analytic Hierarchy Process (AHP) is that all ratio scale nunbers are relative, and are derived form the same underlying fundamental scale used to represent the value judgments of one individual or a group working together on a decision. The arithmetic aggregation rule in a hierarchy makes it unnecessary to normalize the composite results in each level. It is automatically equal to one. Hence, the ratio scale used throughout is the same and not several soales with different multiplicative parameters. There would indeed be confusion if each scale required a different multiplicative parameter for its normalization. Arithmetic aggregation is analogous to dividing a unit disc into pie shaped sectors then treating each sector as the unit whole and dividing it again into subsectors and so on. Calculating the size of each piece a part of the whole is simple. Multiply its relative size in the sector by the relative size of the sector itself; we do not raise it to the power of that sector size as Lootsma's approach suggests. The sum of the pieces obtained is still equal to one. In a hierarchic decision problem, the weighting and aggregation of the priorities of the alternatives yields for each alternative a share of the unit whole assigned to the top goal of the bierarchy. In addition, the sum of the weights of the alternatives measured on all the criteria is automatically egual to one.

Lootsma's choice of geometric aggregation obfuscates the simple idea of dividing the whole into smaller and smaller parts described above and the final result does not automatically sum to one without new normalization. He presents this rule as if it is some kind of an absolute truth, forgetting that multiplication and raising to powers postdate addition and need addition for intuitive justification. Incidentally, there is nothimg mystical about how to comhine the weights of the criteria and alternatives a hierarchy, as there is an infinite number of ways for doing it, but some are less amenable to good intuitive interpretation. The complexity of a mathematical expression does not mean that it is a more accurate representation of the real world. Otherwise, the geometric aggregation rule is itself a first–order approximation to the exponential aggregation rule $s_1^{e^{c_1}-1}$ or to the trigonometric aggregation rule $s_1^{\sin c_1}$, and so on. The purpose of a model should be to clarify and facilitate, not make things more difficult. Let us now examine the use of benefits and costs in conflict resolution.

The same individual can obtain two different ratio scale rankings of the same set of alternatives, one according to benefits and another according to costs (or inverse benefits). The two scales must be combined to enable him to make a decision. He can, for example take their product, or quotient, either of which defines a ratio scale. Obviously one does not obtain a ratio scale by adding two different ratio scales.

With the foregoing in mind, we can address the two technical points Lootsma raises in his note. The first is that the benefits obtained as ratio scale numbers cannot be compared with the costs. Every time we make an important decision, there are benefits to consider and there are costs or pains involved to reap those benefits. Do the benefits justify the costs? Clearly, it can be determined independently that if the costs are negligible, only the benefits are used to justify which alternative to choose, and if the benefits are negligible (valued in pennies for example) but the costs are significant (valued in millions of dollars), then only the costs are used to make the decision. Complexity in the decision arises when the costs and the benefits are of the same order of magnitude and their relative scales are the same. For many practical problems including conflicts this is the real situation. The analysis must take into consideration both, and benefit to cost ratio is one of the simplest functions of the two with prevalent use in society.

Next, in assessing one's perceptions of another person's benefits and costs, one adopts a different value system but with the same philosophy regarding trading off of benefits and costs. Since in calculating a party's index a ratio of sums of products of benefits for one party and costs for the other is taken, the parameter which distinguishes between the two systems of ratios devised by the same individual cancels, and the index for each of the parties is measured in a meningful way.

The second of Lootsma's points has to do with the difficulty of reconciling different value systems. This is indeed an important point which we kept in mind when developing the conflict resolution approach. Where such concern with degrees of freedom is legitimate, we have been very careful not to combine values which arose from the separate ratio scales of opposing parties. There is never a mixing of one party's value system and index with that of another party. It would be naive for one working in the area of conflict resolution (as an OR researcher, teacher of game theory, and as previous analyst at the Arms Control and Disarmament Agency) not to realize that different people have different assessments of a situation, and generally do not have a common unit to relate these assessments. This is generally a big hurdle in any conflict.

In conflict resolution, each party would only examine concessions by the other side which it considers to be worthwhile in trading off. Trivial concessions would not be considered. If the second party has no worthwhile concessions to make, the first party would consider its gains to be zero, and there would be no resolution to the conflict. It is characteristic of negotiations that there may be concessions offered by each side that would be considered a small or a big match for the opponent's concessions. If the number of concessions made by each side is different, the corresponding priorities are weighted by the relative number before normalization so that again benefits and costs can be compared. In sum, each party can in fact compare its benefits and costs and its perceived benefits and costs for the other party within its value system, as parties always do in conflict, without Lootsma's suggestion that different normalization parameters make the task impossible. Conflicts do get resolved after all.

This and several other applications make it crystal clear that each party, for its own understanding, must assess its own values and perceptions of the benefits and costs of the concessions made by it and by its opponent, respectively. Naturally these perceptions would not coincide with the other party's values and perceptions. The results are two separate indices, one for each party. Each index is a ratio of gains to losses derived only from one party's point of view. Even after this, it is never suggested that the two indices be combined as this would indeed violate what we just said. In fact, we only seek feasible packages of concessions for which each party's gains are not less than its losses according to its own index. All such concessions (if any), provide an opportunity for negotiation. A mediator may, by talking to both sides, attempt to identify and trade off one party's values against another's. But in this particular application we did not make such a comparison.

Thus it appears that Lootsma, by not looking carefully at how the ratios are defined, compared the AHP with how he does things and imagined and mis–interpreted the thrust of the work. He makes one think that he does not want the OR community involved in the resolution of the conflict in South Africa, as it may show some inequities he himself does not wish to see coming out in the open. In addition, he apparently feels qualified to be a judge of methodologies for the entire OR community.

Finally, we note that no one is going to use a model that does not work, and is a good criterion to justify the use of a model. Since Lootsma has not offered us any convincing model of his own for conflict resolution, we stand by our approach until it is shown not to work in practice.