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A CASE STUDY IN VALVE DESIGN.

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ABSTRACT

The design of a special type of valve gave rise to a problem which lead the mechanical engineers who designed the valve to call in the help of the author. The final solution to the problem may not be mathematically sophisticated (in fact, it may seem trivial to some readers), but the steps through which the design team struggled and the final approach which resulted in a practically feasible solution may be of interest to other practitioners.

1. INTRODUCTION

The usual applications of valve technology call for infinitely variable valves, where the flow through a valve is regulated by the position of a vane or a plunger. For more details, see Warring [1]. However, some engineering applications demand the use of so-called "digital" valves, which give discrete jumps in flow. The flow through such a digital valve is controlled with the aid of a fixed vane with different sized holes in it (see Figure 1). Each of these holes can be either completely open or completely closed; no inbetween settings are possible. The opening and closing of the holes are effected with solenoid switches which are electronically controlled.

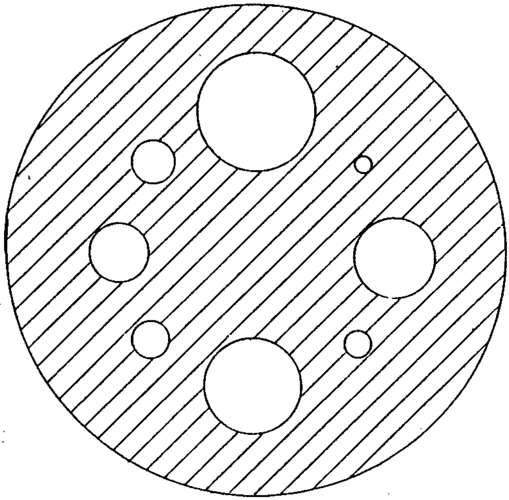

FIGURE 1: Diagram of the fixed vane in a digital valve.

Recently a certain company decided that their specific application called for such a digital valve. Due to reasons of confidentiality the name of the company, as well as the application, must remain a secret. The designers of the valve were faced with the problem of choosing the size of the areas for the holes in the vane of the valve, subject to certain constraints on the performance and control of the válve.

The problem will be set out in greater detail in the next section. In the third section different solution strategies for the problem will be presented, including an approach which led to the final solution. The last section will be devoted to a discussion of implementation issues.

The following notational conventions will be observed in the rest of this article: Real numbers will be indicated by lower case Latin letters (e.g. a_i), sets by upper case Latin letters (e.g. S) and vectors by underlined lower case Latin letters (e.g. $\underline{a}$).

2. A FORMAL DESCRIPTION OF THE PROBLEM

Because of the confidentiality restrictions, symbols will be used to describe the problem and no numerical data will be supplied.

For reasons of control, the number of holes in the vane of the valve was chosen as a fixed number right at the beginning. We shall denote this number by n and assume that its value is known. The desired performance of the valve also prescribed the minimum and maximum total area of the holes in the vane. Denote these areas (also known) by a_{min} and a_{max} respectively.

The designers of the valve identified two desirable qualities for the valve. These qualities will be given here in the form presented by the mechanical engineers in the company. Their "translation" into a form usable for the purposes of modelling and solution will then be discussed.

(1) The smallest "jumps" in the flow through the valve for different settings of the switches should be distributed as evenly as possible throughout the operating range (a_{min} - a_{max}) of the valve.

(2) The "jumps" in the flow between settings of the switches giving flows of consecutive magnitude, should be as small as possible.

The fact that the number of possible settings of the switches allows 2^n-1 different flows through the valve, caused terms such as "combinatorial optimization", "penalty functions", etc. to be bandied about in the group responsible for the design of the valve. At that point they decided to call in a consultant from outside, with the result that the author was approached.

Superficially the problem had all the appearances of a multi-criteria integer programming problem, with the result that the author, like any Operations Researcher worth his/her salt, was only too keen to get involved. Luckily a few factors prevented him from making a complete fool of himself, as will become clear in the next section.

3. SOLUTION STRATEGIES

It was decided by the group (including the author at that stage) to try an approach in which the jumps between the consecutive flows were identical throughout the operating range of the valve. Thus a solution could be obtained which would be optimal for the first objective set in section 2. Without loss of generality the holes can be ordered according to increasing areas.

In order to model this approach, call the difference between consecutive total areas in the vane the **resolution** of the valve and denote it by d. To achieve the results we are trying to model, the areas of holes should be a_{min}; a_1d; a_2d; ... ; $a_{n-1}d$, where the positive integers a_1, a_2, ... a_{n-1} are such that:

$$(a_1 + a_2 + \dots + a_{n-1})d = a_{max} - a_{min} \qquad \text{- - - - - - (3.1)}$$

Any integer between 1 and $(a_1 + a_2 + \dots + a_{n-1})$ can be represented as a sum of some combination of $\{a_1; a_2; \dots a_{n-1}\}$. - (3.2)

Notice that one hole was "reserved" for the minimum area a_{min} to make sure that area was available. See also the discussion in the last section on this point.

It follows directly from (3.1) that

$$d = \frac{a_{max} - a_{min}}{a_1 + a_2 + \ldots + a_{n-1}}$$

Thus, to minimize the resolution d (see objective 2 in section 2), the sum $s = a_1 + a_2 + \ldots + a_{n-1}$ should be maximized. The problem can thus be formulated as the following problem (P):

Maximize $s = a_1 + a_2 + \ldots + a_{n-1}$
Subject to:
$\underline{a} \in X$,

where the vector $\underline{a} = [a_1 \; a_2 \; \ldots \; a_{n-1}]$ and X is the set of all integer vectors satisfying (3.2).

Three sets of values for $\{a_i\}$ were suggested, the first one by an engineer in the design group and the other two by the author. These three suggestions will be discussed in turn.

3.1 An obvious solution:

The easiest solution would be to divide the interval $[a_{min}; a_{max}]$ into equal subintervals. Thus we have $a_i = i$. The resulting resolution will then be

$$d = 2(a_{max} - a_{min})/[n(n-1)].$$

It turned out not to be a good enough resolution in practice.

3.2 A solution based on the "change" problem:

A solution which promised to be an improvement on the previous one was based on the type of solution proposed for the so-called "money-changing" problem (e.g. see Wilf [2]). Consider the sequence of integers

1; 2; 2; 5; 10; 20; 20; 50; 100; 200; 200; 500; ...

It is well known that this sequence satisfy constraint (3.2). It is also easy to show that the resolution will improve with this approach for $n > 5$. Even though this solution to (P) is an improvement over the obvious one above, it is clear that it still contains a degree of redundancy in the sense that some integers can be represented by more than one combination of the given sequence. For example, 5 can be represented by 1+2+2, as well as 5 on its own. The elimination of such redundancy may lead to a series which grows quicker than the present one. Thus the objective of (P) could perhaps be improved by choosing a series without such redundancy. In a sense the optimal solution should be one without any redundancy in the sense described above. The next section will discuss such a series.

3.3 A solution based on partitions of integers:

The developments outlined in the previous section brought the realization that the solution to (P) should be sought with the aid of number theoretical concepts. The particular concepts necessary for our discussion (one definition and one theorem) will be collected from Roberts [3].

Definition: A partition of the nonnegative integer s into n parts is a set of nonnegative integers p_i, $(i = 1, \ldots n)$, such that

$$s = p_1 + p_2 + \ldots + p_n.$$

Thus we see that (P) can be reformulated as follows:

> Find a partition with n parts of s, such that it will also be a partition of all integers between 1 and s and that it will maximize s.

As far as the maximization of s is concerned, we have argued above that this would mean, at the very least, that no redundancy (in the sense identified above) should be present in the partiion. One way in which to ensure nonredundancy is to make sure that each number between 1 and s should have a unique partition. At this point a result from number theory comes into play again:

Theorem: (Roberts [3, p172])

Any integer can be uniquely partitioned into binary numbers.

This means that the partion $a_i = 2^{i-1}$, $(i = 1, 2, \ldots n-1)$ will in some sense be an optimal solution for (P). The resolution for this partition is

$$d = (a_{max}-a_{min})/(2^n-1)$$

It can be shown with the aid of some elementary mathematics that the sequence used here grows much more rapidly than the previous one. Thus a much better resolution results from this partition. It was found in practice that the resulting resolution is so good that it could scarcely be measured physically. So, even if this solution for (P) is not demonstrably optimal, it is good enough for practical purposes.

When this solution was shown to a computer scientist, he described it as trivial. This may be true as seen from the point of view of someone who thinks in binary terms in any case. But reaching the point where the problem formulation became accessible to that way of thinking, took a lot of effort, as seen above. Moral of the story: Although the final solution may be trivial, the process through which it was found is not trivial. Many other such examples exist in the Operations Research literature, e.g. Woolsey [4].

4. IMPLEMENTATION

In conclusion, two points regarding the implementation of the proposed solution deserve mention. Firstly, given the elementary nature of the solution, it was very easy to convince the design engineers to accept and implement the final solution in principle.

Secondly, the resolution resulting from this solution will of course improve dramatically if more holes can be constructed. As was mentioned at the beginning, the number of holes is constrained by the method of control of the valve. The solution generated above makes provision for one hole the size of a_{min} and the rest of the holes are then constructed according to the binary partition. If the hole reserved for a_{min} is also constructed as one of the "binary" holes, the sequence length increases by one and the resolution improves accordingly. One must then make sure that at least one of the resulting combinations of areas approximate a_{min} closely enough. For the real world problem the resolution was so good that the difference between a_{min} and the closest approximation could not even be measured in practice.

5. ACKNOWLEDGEMENTS

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