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# NETWORK MODELS FOR THE EXPANSION OF GRAIN STORAGE FACILITIES

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## ABSTRACT

This study was undertaken for a number of grain co-operatives from a certain region in South Africa. A previous study done for these co-operatives indicated that existing storage facilities should be extended to accommodate increased production in the region. The proposed extension plan recommended the phased construction of extensions to silos in the region. These extensions should have a predetermined total storage capacity. The questions that remained were: How many silos should be extended, which silos should be extended and what should the capacity of each extension be? The objective is to minimize the sum of construction costs and the total cost of transportation between farmers and silos. A linear mixed integer programming model can be used to solve this problem. However, given the computational facilities at the disposal of the researchers, it was decided to rather use a heuristic procedure based on a transshipment model. These network models, their solution and the recommendations made on the strength of those solutions will be discussed.

## 1. INTRODUCTION

The total storage and transportation cost of grain during its distribution from producer to consumer is influenced by the number and location of storage facilities. The transportation cost from the producer to the silo plus the equivalent annual component of the construction cost of the silo amount to approximately 55% of the total annual cost of distribution (Du Preez [1]). It is thus very important to minimize these costs.



The bigger the capacity of the silo, the lower the unit storage cost of grain. But a storage facility with a big capacity has a negative influence on the total transportation cost from the producers to the facility. The problem is thus to find the number, location and capacity of storage facilities which will result in the most cost-effective storage system satisfying the demand for storage capacity for a specified region and a specified production level.

The project was done for a number of grain co-operatives from a certain region in South Africa. This region had at its disposal a number of storage facilities. Therefore, the client was concerned with the extension of existing facilities (in the form of extra bins) and not the construction of new silo complexes. However, the approach presented in this paper can also be used in the latter case. The extra storage capacity needed for the region was determined in a previous study for the client. A discussion of the forecasting methods used in this study falls outside the scope of this paper. In the sequel we shall assume that the required total extra storage capacity is known. The only questions addressed in this paper are: How many extensions should there be? Where should these extensions be located and what should their capacities be (the total capacity should of course be equal to the required capacity previously determined)?

## 2. THE FORMAL PROBLEM

As mentioned in the introduction, the study was concerned with the minimization of construction and transportation costs. A few other assumptions were made:



(a) The decision maker identified a number of possible locations for extension before the formulation of the problem - usually those existing facilities currently experiencing the biggests bottlenecks.

(b) The total required extension capacity was known for the region.

(c) Basic data required were available, for example costs, distances, etc (see the formulation below).

Before a mathematical formulation of the problem can be given, the necessary notation must be introduced:

VARIABLES:

$x_{ij}$ = number of units of grain transported from producer i (about 190 of them) to facility j (up to 12 of them);

$$z_j = \begin{cases} 1 & \text{if facility } j \text{ is extended;} \\ 0 & \text{otherwise;} \end{cases}$$

$y_j$ = capacity of the extension at facility j.

CONSTANTS:

$u_j$ = upper limit on extension at facility j;

$k_j$ = existing capacity at facility j;

$q_i$ = production of producer i;

$c_{ij}$ = cost of transporting one unit from producer i to facility j;



$f_j$ = equivalent annual fixed construction cost for extension at facility j;

$v_j$ = equivalent annual variable construction cost per unit of extension at facility j;

U = total extra storage capacity required for the region;

K = number of extensions considered.

SETS:

J = set of indices indicating facilities where extension is possible ($|J| > K$).

The problem can be formulated as follows:

Minimize $\Sigma_i \Sigma_j c_{ij} x_{ij} + \Sigma_j (f_j z_j + v_j y_j)$

Subject to:

$$\Sigma_j x_{ij} = q_i, \text{ for all } i \quad \text{- - - - - - - -} \quad (1)$$

$$\Sigma_i x_{ij} \le k_j, \text{ for all } j \text{ not in } J \quad \text{- - -} \quad (2)$$

$$\Sigma_i x_{ij} \le k_j + y_j, \text{ for all } j \text{ in } J \quad \text{- -} \quad (3)$$

$$\Sigma_j y_j = U \quad \text{- - - - - - - - - - - - - - -} \quad (4)$$

$$y_j \le u_j z_j, \text{ for all } j \text{ in } J \quad \text{- - - - - -} \quad (5)$$

$$\Sigma_j z_j = K \quad \text{- - - - - - - - - - - - - - -} \quad (6)$$

$x_{ij} \geq 0$ for all i and j.
$y_j \geq 0$ for all j in J.
$z_j = 0/1$ for all j in J.



This linear mixed integer programming problem had, for the practical problem, 2275 $x_{ij}$-variables and up to 12 each of the $y_j$ and $z_j$ variables. The computing facilities (microcomputers only - the available mainframe did not have the necessary software) and software (MILP88) available could not handle problems of this size and complexity. Also keep in mind that the problem had to be solved a few times with different sets (J) and constants (K) to determine the optimal number of extensions necessary. Each of these problems differed from the previous one in a number of variables and constraints, making the approach cumbersome and computationally expensive. It was therefore decided to approach the problem in a heuristic fashion, making use of network models.

## 3. NETWORK MODEL FOR THE PROBLEM

In practice it was necessary to make a few simplifying assumptions in order to arrive at a solution to the problem. In the first instance, it was found that it would be very expensive (and probably not cost effective) to gather the data to measure the differences in fixed ($f_j$) and variable ($v_j$) costs for the different possible extensions in the same region. It was thus assumed that these costs were equal. In section 4 we shall discuss the possibility of using the present approach, while taking the differences in construction costs into account. A previous study conducted at the Institute for Industrial Engineering (Du Preez [1, p

B.51]) quantified these unit costs of extensions for different capacities. (See Figure 1).



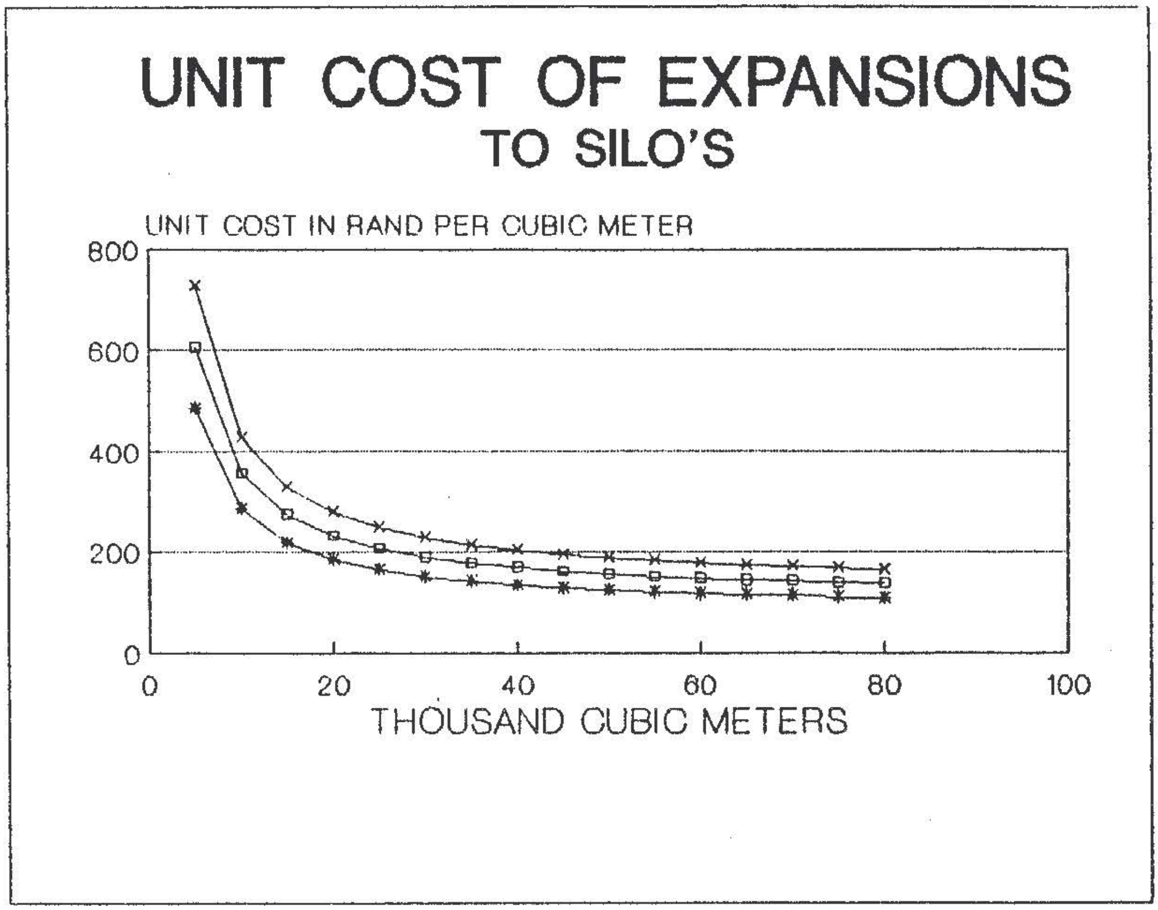


FIGURE 1: Unit cost of extensions to silos

This assumption had the result that the objective function of the model in the previous section becomes independent of $y_j$ and $z_j$ (see constraints (4) and (6)). The resulting model thus minimizes only the total transportation cost. The construction costs could be calculated independently and added after the optimization to obtain the total cost.

The variables $y_j$ and $z_j$ still played a role in the optimization by constraining the capacities of some of the facilities. These variables complicated the optimization process. A heuristic procedure was designed to circumvent this complication.

The procedure begins by allowing only one extension. A few likely locations are identified beforehand as candidates for extension. The optimization is then repeated with the candidate extensions considered one by one. The optimization is done with the aid of the network model to be discussed shortly. The total transportation and construction costs are calculated as discussed above and the best candidate chosen for further consideration. The procedure is repeated with two extensions allowed. Again, a few likely combinations of two locations are identified beforehand. The transportation cost for the best combination is compared to the transportation cost for one extension. The difference (savings) in transportation cost is now capitalized using a real interest rate and compared with the additional capital investment required to construct the one additional extension. The procedure continues in this manner by allowing one more extension during each iteration, each time testing whether a more cost effective solution has been obtained. The procedure stops when the savings in transportation costs is less than the additional construction cost. A flow diagram of the heuristic procedure is shown in Figure 2.

It is obvious that the quality of the solution generated by the heuristic will depend upon the choice of possible extension locations during each iteration. As a general rule the existing locations currently experiencing the greatest bottlenecks were used in order of their capacity shortage. However, some locations specified by the client were also added to the candidate list. In order to measure the quality of the solutions obtained, a basis for comparison was calculated as follows: The total extension capacity was assumed to be unlimited. Each of the producers was then allocated to the facility closest to him. The resulting total transportation cost is a lower bound on the optimal value of our problem. The results obtained through the heuristic procedure showed an increase of less than 5% over this lower bound.

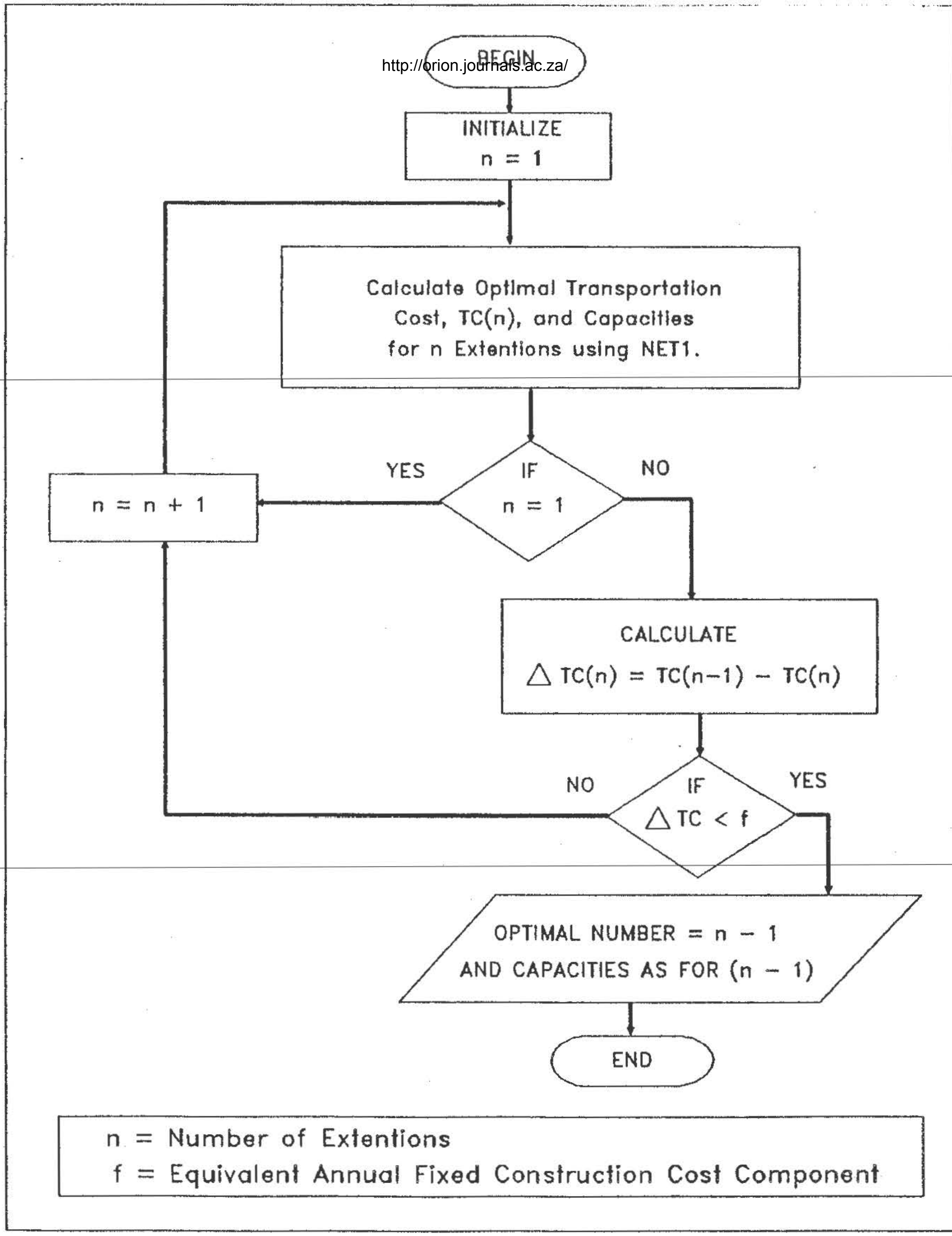




FIGURE 2: Diagram of heuristic procedure

The beauty of this approach lies in the fact that each new problem could be found from the previous one by changing only a few arcs of a network. The software used facilitated this approach.



The optimization phase in the heuristic procedure was approached by formulating the network problem in figure 3. The i-nodes represent producers; the j-nodes the existing facilities; the leftmost node (189) is a super-source; the node at the bottom (192) is a super-sink; the node in the top right-hand corner (190) is a sink representing flow to the facilities which fill up existing capacity, and the node in the bottom right-hand corner (191) is a sink representing extension capacities at existing facilities. Notice that in this case $K = |J|$. Both the super-source and the super-sink were required by the solution method and do not play any meaningful role in the formulation. Each arc is labeled with a triplet of the form ($u_p$ ; $l_p$ ; $c_p$), where $u_p$ is the upper bound on, $l_p$ the lower bound on and $c_p$ the unit cost of flow through the arc. The arcs connecting the super-source with the i-nodes ensure that exactly the production of the producers represented by the i-nodes flow into, and thus by conservation of flow also out of, these nodes (constraint (1) in the linear mixed integer programming problem in the previous section). No upper bounds were placed on flow from producers to facilities (the arcs connecting the i- and j-nodes), but the transportation costs were included in the triplets for these arcs. The arcs connecting the j-nodes and the nodes representing existing and extension capacity are indicated by solid and double lines respectively. The upper limits on the solid arcs represent the existing capacities at the facilities (constraint (2) above), whereas the upper limits on the double arcs represent the extension capacities (constraints (3) and (5) above). Note that there will only be K double arcs. Constraint (4) above is accounted for by the capacity on the arc joining the node representing the extension capacity to the super-sink. Constraint (6) is accounted for by the fact that the number of candidate locations for extension equals the number of extensions considered ($K = |J|$).



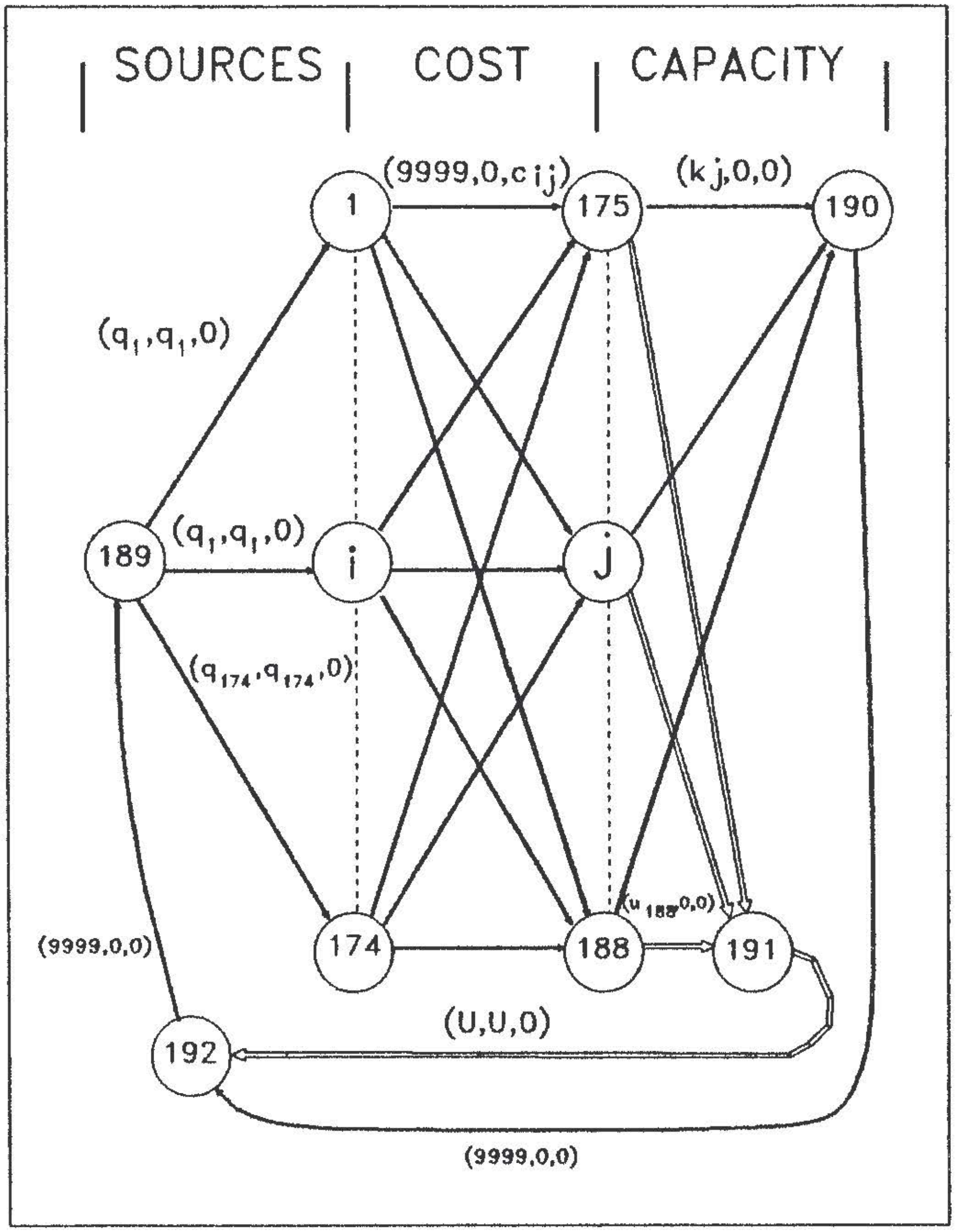


FIGURE 3: The network model

The network problems were solved with the NET1 program (Du Preez, et al [2]). The user friendliness of this software was an important factor in the success of the study.

## 4. RESULTS OF THE SOLUTION OF THE MODEL

The results of the approach described above will be discussed for one set of alternatives investigated. The example was concerned with a total extension capacity of 85 000 $m^3$. The alternatives investigated were for one, two, and three extra silo(s) at those facilities experiencing the biggest bottlenecks at the present stage. For each alternative the best total transportation cost was calculated as described in the previous section. These costs were then evaluated as described in the previous section to obtain a good solution.



The results for one extension are summarized in Table 1. Obviously facility 1 must be extended if only one extension is allowed. Similar results for the extension of two facilities are summarized in Table 2. Again it is obvious that the best combination is extension at facilities 1 and 2. Moreover, the combination of two silos results in savings of R 321 000 p.a. over one extension. Without going into the rest of the details, it can be shown with this approach that three extensions cannot bring about enough savings in transportation cost to justify the extra cost of an additional extension.

| Location | Transportation cost p.a. (million Rand) |
|---|---|
| Facility 1 | 5,109 |
| Facility 2 | 5,36 |
| Facility 3 | 5,373 |
| Facility 4 | 5,497 |

TABLE 1: Transportation cost for one extension

| Combination of two locations | Capacity ($m^3$) | Transportation cost p.a. (Million Rand) |
|---|---|---|
| Facility 1 | 28 464 | 4,788 |
| Facility 2 | 56 536 | |
| Facility 1 | 34 098 | 4,828 |
| Facility 5 | 50 902 | |
| Facility 4 | 45 435 | 4,902 |
| Facility 5 | 39 565 | |
| Facility 2 | 39 764 | 4,936 |
| Facility 4 | 45 236 | |

TABLE 2: Transportation cost of combination of two facilities

An additional benefit of the approach experienced during the use thereof is that the annual allocations of producers to silos can now be made using existing data structures. Savings in this respect can be as much as 17% of transportation cost. Of course, some data such as production figures will have to be updated annually if the allocations are to be made in similar fashion.

It is obvious that, had the values of the fixed ($f_j$) and variable ($v_j$) construction costs been available, they could have been used without any difficulty in the present approach. The original assumption could thus have been watered down to a certain extent.

Typical solution times for the approach depend on the type of computer used and the amount of preprocessing of data necessary. The actual execution time for the example was less than 30 minutes on a 16 MHz 386 IBM compatible microcomputer.



## 5. ACCEPTANCE OF RESULTS BY THE CLIENT

"Selling" the results to the client was the last step in the project. The results were presented in a substantial report and the first author attended a meeting where the managers of all the facilities in the region were present. Three of the four managers were happy with the recommendations, but the fourth one balked at the fact that the results did not support his claim to extension at his own facility.

The fact that such a large proportion of the managers accepted the results at the first opportunity was surprizing. Before this study was initiated, they wanted to extend eight of the existing silos. The recommendation of the report was that only three extensions be done. Given the large amount of vested interest in the extension of a facility, more dissent was expected.

The single dissenting manager tried to query the results on the grounds that the data used were inaccurate. The first author offered to sit down with him, update the data and rerun the program, well knowing that the results would not change to such an extent that his proposals would become acceptable. What was agreed upon at the meeting, was that one more run with the program will be done with the recommendations of the dissenting manager reflected in the data. The penalty cost of implementing his recommendations could thus be calculated.

The preparations were made for the new run. The day the run was completed, a communication was received from the dissenting manager stating that he had changed his mind and had accepted the results as valid. No doubt the fact that the recommendations of the report would save the industry R8 million played a significant role in his decision.



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