*ORiON, Vol. 9, No. 1, pp. 1-12* *ISSN 0259-191-X*

# A SIMULATION OF A PROPOSED THREE-PHASE METHOD FOR PROPORTIONAL REPRESENTATION IN SOUTH AFRICA

G. GELDENHUYS, M.J. SCIOCATTI and P.DU T. FOURIE

**Department of Applied Mathematics, University of Stellenbosch, 7600 Stellenbosch, Republic of South Africa**

**ABSTRACT :**

In October 1992 the President's Council of the Republic of South Africa approved a report on a proportional polling system for the country in a new constitutional dispensation. A three-phase method is proposed to ensure that there is not only proportionality with respect to the electoral regions of the country, but also on a party basis with respect to the votes cast in the election. A lower house of parliament with 400 seats is proposed. In the first phase 300 of the seats are made available to the various regions in proportion to the number of eligible voters in the respective regions. In the election the various parties compete for these 300 seats in the different regions. The second phase consists of allocating the seats to the parties on the basis of the actual votes cast for them in the regions. Because of factors such as variable percentage polls and support for the parties in the separate regions, it could happen that a particular party's portion of the 300 seats is not in accordance with the votes that it receives nationally. In the third phase the remaining 100 seats are used to rectify such situations. On the basis of the votes cast, these 100 seats are used for compensatory purposes, so that the final allocation of the 400 seats to the parties should be proportional to the support for the parties in the election. Fixed regional party lists of candidates for the election and the Jefferson allocation method are used in the applicable phases. We translate the prose of the President's Council report into formal mathematical descriptions of the proposed methods. Several hypothetical examples are used to illustrate the methods and to point out possible problems. A computer program which implements the methods is described briefly and is used to simulate various elections. These simulations show that 100 compensatory seats should be sufficient for the purpose for which they were introduced. We hope that our descriptions and analyses will contribute to the debate on an acceptable and practical electoral system in a new South Africa.

## 1. INTRODUCTION

South Africa has a long history of representative government. During the period 1852-1857 the two British colonies, the Cape Colony and Natal, as well as the South African Republic (Transvaal) and the Republic of the Orange Free State, obtained elected assemblies with various degrees of independence. After the South African War of 1899-1902 the two republics lost their independence and British rule was reimposed. The South Africa Act, approved by the British parliament in 1909, led to the formation of the Union of South Africa, consisting of four provinces with a degree of autonomy.

The political history of the Union - since 1961 the Republic - of South Africa is so well-known, at least in outline, that it seems unnecessary to repeat it here. Suffice it to say that it culminated in the release of Mr Nelson Mandela from prison and the unbanning of various political organizations in February 1990. Since then South Africa has been moving inexorably towards elections for a multiracial government. This has been a turbulent process involving conflict on many levels. Although garnering less attention than topics like constitutional and economic planning, the election process itself has been an important subject in negotiations between the various groups. In December 1991 President FW de Klerk therefore asked the President's Council of the Republic of South Africa to prepare a report on various alternatives for a polling system under which elections might be held. (The President's Council is an indirectly elected body which has assumed some of the functions of the second chamber, the Senate, which was abolished with the institution of the tricameral Parliament in 1984.)

In October 1992 the President's Council approved a report on a proportional polling system for South Africa in a new constitutional dispensation [5]. The purpose of this paper is firstly to put the verbal descriptions in the report into a mathematical form, secondly to illustrate the proposed method by means of numerical examples, thirdly to test the feasibility of the method, and fourthly to provide a few references to the literature not mentioned in [5] which will probably be more accessible and relevant for operations researchers. It is hoped that the paper will contribute to the debate on an acceptable and workable polling system for South Africa.

The paper is structured as follows. The notation used is explained in Section 2. The polling system accepted by the President's Council constantly makes use of the so-called "Jefferson method"; this is briefly described in Section 3. In Section 4 it is explained how the proposed polling system uses different phases to make provision for proportional representation on a regional basis as well as on a party-political basis. Hypothetical examples to illustrate the method and to point out possible pitfalls are discussed in Sections 5 and 6. A computer program which can be used to simulate the effect of the system, and to test its stability under varying conditions, is briefly described in Section 7. The results of a wide variety of test runs are discussed in Section 8, and our conclusions are given in Section 9. Finally a brief bibliography is supplied.

## 2. NOTATION

The following notation is used in the description of the method and in the examples. For real $x$

$$\lfloor\lfloor x \rfloor\rfloor = \lfloor x \rfloor \text{ if } x \text{ is not an integer},$$
$$= x \text{ or } x-1 \text{ if } x \text{ is an integer}$$

The simbol $\lfloor x \rfloor$ represents the largest integer less than or equal to $x$. The second part of the notation for $\lfloor\lfloor x \rfloor\rfloor$ above is only needed in special cases, as will be shown by means of examples in Section 6.

## 3. THE JEFFERSON METHOD APPLIED TO REGIONS

The methods recommended in [5] for the allocation of seats are known as the methods of d'Hondt and Hagenbach-Bischoff. In [3,4] these two methods are described as being equivalent to the so-called Jefferson method. As the Jefferson method is well known in the OR literature [1,2,3], we shall in the rest of this paper only refer to this method.

Let

$$\mathbf{p} = (p_1, p_2, \ldots, p_s)$$

be the vector of (voter) populations for $s$ regions, with total (voter) population for South Africa

$$p = \sum_{i=1}^{s} p_i .$$

Let the lower house of Parliament (i.e. the House of Commons in the UK or the House of Representatives in the USA) be of size h, in other words h seats will be contested. The suggestions of the President's Council as to how these h seats may be allocated in three phases will be discussed in Section 4. Here we illustrate the Jefferson method by describing how it could be applied to allocate the h seats directly (in one phase) to the different regions.

First the average (voter) population per seat for the whole country is calculated:

$$\lambda* = \frac{p}{h} .$$

Then a number $\lambda$ is chosen,

$$0 < \lambda \leq \lambda* ,$$

with the property that

$$\sum_{i=1}^{s} \left\lfloor \left\lfloor \frac{p_i}{\lambda} \right\rfloor \right\rfloor = h .$$

The number of seats in region i is

$$a_i = \left\lfloor \left\lfloor \frac{p_i}{\lambda} \right\rfloor \right\rfloor , \; i = 1, 2, \ldots, s,$$

so that

$$\sum_{i=1}^{s} a_i = h .$$

This gives the Jefferson allocation

$$\mathbf{a} = (a_1, a_2, \ldots, a_s) .$$

The chosen number $\lambda$ is not necessarily unique. In the so-called Droop method [5] it is recommended that

$$\lambda = \frac{p}{h+1} + 1$$

be chosen, but this choice is not necessarily correct, as will be shown in Sections 5 and 6. If it is not correct, further calculations must be done to allocate the correct number of seats [5]. We think that it is probably simpler to calculate a correct $\lambda$ by inspection or an iterative search procedure, and then to do a correct allocation using this $\lambda$. We follow this course in the examples which we discuss later.

The normal (single-phase) Jefferson method has a number of properties which make it an attractive choice as an allocation method. For example, it is house monotone [1], meaning that no region will lose a seat if the size of the house increases. It also encourages the formation of coalitions [2]. These and other properties are discussed fully in [3].

## 4. THE PROPOSED METHOD

The method proposed in the report of the President's Council [5] has three phases. The purpose is to ensure not only that proportional representation will be achieved on a regional basis in terms of the number of eligible voters, but also that parties will be represented in the house in proportion to the total number of votes they attract.

### 4.1 THE FIRST PHASE: PRELIMINARY ALLOCATION OF SEATS TO REGIONS

In the first phase a smaller house size $h_1$ is used $(h_1 < h)$. Before the election a *preliminary* or *partial* allocation is made of seats to regions using the Jefferson method and the (voter) population vector, **p**:

$$\mathbf{b} = (b_1, b_2, \ldots, b_s)$$

where

$$\sum_{i=1}^{s} b_i = h_1 .$$

In the election the parties compete directly for these seats in the various regions.

### 4.2 THE SECOND PHASE: ALLOCATION OF REGIONAL SEATS TO PARTIES

Suppose there are m parties country-wide and that each party competes in every region. (If a party is not represented in a region, its vote total can be taken as 0.) Suppose in region i the election result is

$$\mathbf{v}^i = (v_1^i, v_2^i, \ldots, v_m^i),$$

with

$$v^i = \sum_{j=1}^{m} v_j^i$$

the total number of votes cast in the region. In the second phase the $b_i$ provisional seats of a region i are allocated to the parties on the basis of the regional results and with the Jefferson method according to the vector

$$\mathbf{c}^i = (c_1^i, c_2^i, \ldots, c_m^i),$$

where

$$\sum_{j=1}^{m} c_j^i = b_i .$$

Because the voting percentages and the relative strengths of the political parties in the various regions may vary widely, a party may obtain a share of the $h_1$ seats not proportional to its actual number of votes country-wide. The third phase is used to counteract this possible distortion.

## 4.3 THE THIRD PHASE : COMPENSATION

The third phase proceeds as follows. Country-wide party j obtains

$$d_j = \sum_{i=1}^{s} v_j^i$$

votes. This leads to a vote vector

$$\mathbf{d} = (d_1, d_2, \ldots, d_m),$$

where

$$d = \sum_{j=1}^{m} d_j$$

is the total number of votes cast in the entire country.

The Jefferson method is used in conjunction with the vote vector to make an ideal party allocation of h seats,

$$\mathbf{g} = (g_1, g_2, \ldots, g_m),$$

where

$$h = \sum_{j=1}^{m} g_j .$$

This is the allocation which should be made in the house of size h to the various parties to achieve proportional representation on a party basis.

In the preliminary allocations $\mathbf{c}^i$, $i =$ 1, 2, ..., $s$, of the $h_1$ seats to the various parties in the various regions, party $j$ receives altogether

$$t_j = \sum_{i=1}^{s} c_j^i$$

seats country-wide, with

$$\sum_{j=1}^{m} t_j = h_1 .$$

This results in a vector, the *preliminary party allocation* of $h_1$ seats,

$$\mathbf{t} = (t_1, t_2, \ldots, t_m).$$

We now investigate the difference between the ideal party allocation **g** and the preliminary party allocation **t**:

$$\mathbf{u} = \mathbf{g} - \mathbf{t} .$$

The differences are now taken into account by using the remaining

$$\Delta h = h - h_1$$

seats, the so-called *compensating seats*.

If $u_j \leq 0$, nothing further is done for party $j$ and it receives its $t_j$ seats in the final allocation.

If $u_j > 0$, compensation is applied from the $\Delta h$ seats to allocate $\Delta t_j$ additional seats to $j$, where

$$\sum_{j \text{ so that } u_j > 0} \Delta t_j = \Delta h .$$

The final party allocation is then

$$\mathbf{F} = (f_1,\ f_2,\ \ldots,\ f_m),$$

where

$$\sum_{j=1}^{m} f_j = h$$

and where (for $j = 1,2, \ldots, m$))

$$\begin{aligned} f_j &= t_j + \Delta t_j \quad \text{if } u_j > 0 \ , \\ &= t_j \qquad\qquad \text{if } u_j \le 0 . \end{aligned}$$

It is clear that problems will arise if $u_j < 0$ for a party $j$, because that party will receive more seats in the smaller "house" of $h_1$ than the number to which it is entitled in the house of h seats.

## 4.4 COMPENSATION TAKING REGIONAL ASPECTS INTO ACCOUNT

In the proposals of the President's Council measures are taken to ensure that the $\Delta t_j$ compensating seats for party $j$ also take regional aspects into account. For example, this may be done by applying the Jefferson method to the "population vector"

$$\mathbf{w}^j = (v_j^1,\ v_j^2,\ \ldots,\ v_j^s)$$

of votes cast in the various regions for party $j$, "population" $d_j$ of total votes cast for party $j$ and "house size" $f_j$ of ideal seats for party $j$. This will have the effect that party $j$ in region $i$ ideally should receive $\eta_j^i$ seats (proportional to $v_j^i$), where

$$\sum_{i=1}^{s} \eta_j^i = f_j .$$

This also means that the $\Delta t_j$ compensating seats for party j will be subdivided so that region i receives $\Delta t_j^i$ of the $\Delta t_j$ compensating seats, where

$$\Delta t_j^i = \eta_j^i - c_j^i \ , \ i = 1,\ 2,\ \ldots,\ s,$$
$$j = 1,\ 2,\ \ldots,\ m,$$

and

$$\sum_{i=1}^{s} \Delta t_j^i = \Delta t_j \ , \ j = 1,\ 2,\ \ldots,\ m.$$

According to the proposals of the President's Council, each party in each region will provide a list of its candidates in order of preference. In region $i$ the first $\eta_j^i$ candidates on the list of party $j$ will receive seats, $i = 1,2, \ldots, s, j = 1,2, \ldots, m$ . The result will hopefully be that the members chosen will have close links with the regions that they represent.

From a mathematical point of view, this aspect of the proposed method presents no problems . In the examples of Sections 5 and 6 no further attention will therefore be paid to this aspect.

### 4.5 WHAT SHOULD THE SIZE OF COMPENSATION BE?

Generally speaking the problem is as follows:

Given $h$ and $\mathbf{p}$, how large should $h_1$ (and therefore $\Delta h$ ) be to ensure that $\mathbf{F} = \mathbf{g}$ and $\mathbf{u} \geq \mathbf{0}$?

In the proposals of the President's Council

$$h = 400\,,$$
$$h_1 = 300\,,$$
$$\Delta h = 100\,,$$

and examples are used where $s = 9$.

## 5. EXAMPLES

In the examples given below the regions are taken to be the four provinces; this is done purely for convenience. It is probable that in the coming election there will be between nine and eleven regions, more or less coinciding with the nine economic development regions as defined at present [5, p.69].

Example 1

Consider the hypothetical case of four provinces with the numbers of qualified voters and votes cast (in millions) for five parties, A, B, C, D and E, as in the Table 1.

| | | Votes cast (in millions) | | | | | |
|---|---|---|---|---|---|---|---|
| | Voters (millions) | A | B | C | D | E | TOTAL |
| Cape | 5 | 0,28 | 0,84 | 1,40 | 0,56 | 0,42 | 3,50 |
| Natal | 4 | 0,24 | 0,24 | 0,60 | 0,72 | 0,60 | 2,40 |
| Orange Free State | 2 | 0,08 | 0,24 | 0,40 | 0,56 | 0,32 | 1,60 |
| Transvaal | 9 | 1,05 | 0,55 | 1,25 | 0,65 | 1,00 | 4,50 |
| TOTAL | 20 | 1,65 | 1,87 | 3,65 | 2,49 | 2,34 | 12,00 |

**Table 1 : Example of voters and votes cast.**

Note that there are different voting percentages in the different regions, with a country-wide voting percentage of 60%.

Phase 1: In the first phase a certain part of the seats is allocated to the regions . For a house size $h_1 = 300$ the regions (in the order Cape, Natal, Orange Free State and Transvaal) are entitled to the allocation

$$\mathbf{b} = (75, 60, 30, 135)\,.$$

These are the preliminary seats for which the parties compete.

Phase 2: The preliminary regional allocation $\mathbf{b}$ may be made to the parties on the basis of the election results by means of the Jefferson method, as shown in Table 2. For Cape and Natal their respective values for $\lambda^*$ may be used directly. The values

$$\lambda_{\text{OFS}} = 50\ 000$$

and

$$\lambda_{\text{Transvaal}} = 32\ 750$$

were used in the calculations.for the other two provinces.

| | A | B | C | D | E | TOTAL |
|---|---|---|---|---|---|---|
| Cape | 6 | 18 | 30 | 12 | 9 | 75 |
| Natal | 6 | 6 | 15 | 18 | 15 | 60 |
| Orange Free State | 1 | 4 | 8 | 11 | 6 | 30 |
| Transvaal | 32 | 16 | 38 | 19 | 30 | 135 |
| TOTAL | 45 | 44 | 91 | 60 | 60 | 300 |

**Table 2 : Allocation of seats to parties.**

The preliminary allocation t of $h_1 = 300$ seats to parties is therefore

$$\mathbf{t} = (45, 44, 91, 60, 60)\ .$$

Phase 3: If the votes cast are considered country-wide, the number of votes per constituency (for a house size $h = 400$) is

$$\lambda^* = \frac{12 \times 10^6}{400} = 30\,000\,.$$

With $\lambda = 29800$ the Jefferson method yields the ideal party allocation **g** (in the order A, B, C, D, E) for h = 400 seats , where

$$\mathbf{g} = (55, 62, 122, 83, 78)\ .$$

Any value of $\lambda$ in the interval

$$29\,682\,\tfrac{34}{63} < \lambda \leq 29\,918\,\tfrac{2}{61}$$

would in this case have yielded a valid allocation. The value used in the Droop method ,

$$\lambda = \frac{12 \times 10^6}{401} + 1 = 29\,926{,}187\ ,$$

would in this case not yield the correct value.

The compensating seats should be

$$\mathbf{u} = \mathbf{g} - \mathbf{t} = (10,\ 18,\ 31,\ 23,\ 18)$$

and since the components of u are non-negative and their sum is exactly 100 , compensation is feasible in this case.

The $\Delta t_A = 10$ compensatory seats for party $A$ can for example be allocated to the regions as follows. For the ideal "house size" of $f_A = 55$ seats for party $A$ and the country-wide vote total of $d_A$ = 1,65 million votes for $A$, the number of votes for $A$ per seat equals

$$\lambda^* = \frac{1,65 \times 10^6}{55} = 30\,000.$$

With the choice $\lambda_A = 29\,000$ and "population vector"

$$\mathbf{w}^A = (0{,}28;\ 0{,}24;\ 0{,}08;\ 1{,}05) \text{ million}$$

the allocation

$$(\eta_A^1,\ \eta_A^2,\ \eta_A^3,\ \eta_A^4) = (9,\ 8,\ 2,\ 36)$$

of A's 55 seats to the various regions is obtained. Compared to the preliminary allocation in Table 2,

$$(c_A^1,\ c_A^2,\ c_A^3,\ c_A^4) = (6,\ 6,\ 1,\ 32),$$

this means that the 10 compensatory seats for party A are awarded to the regions according to the allocation

$$(\Delta_A^1,\ \Delta_A^2,\ \Delta_A^3,\ \Delta_A^4) = (3,\ 2,\ 1,\ 4)$$

Example 2

One of the problems that may arise is when a party attracts an exceptionally large proportion of the votes in a region with exceptionally many voters , but in which an exceptionally low voting percentage is recorded.

Suppose for example in Table 1 that all the details for the Cape, Natal and Orange Free State remain the same, but that in Transvaal a voting percentage of only 38% is attained, that party A receives 80% and each of the parties B, C, D and E only 5% of the votes cast. The last two rows of Table 1 are changed as shown in Table 3 .

| | Voters | A | B | C | D | E | TOTAL |
|---|---|---|---|---|---|---|---|
| Transvaal | 9 | 2,736 | 0,171 | 0,171 | 0,171 | 0,171 | 3,42 |
| TOTAL | 20 | 3,336 | 1,491 | 2,571 | 2,011 | 1,511 | 10,92 |

**Table 3 : An exceptional voting pattern in Transvaal**

On the basis of the votes cast in Table 3 the following ideal allocation of the $h = 400$ seats to the respective parties A, B, C, D and E must be made:

$$\mathbf{g} = (123,\ 54,\ 94,\ 74,\ 55).$$

The last two rows of Table 2 are modified as in Table 4.

| | A | B | C | D | E | TOTAL |
|---|---|---|---|---|---|---|
| Transvaal | 111 | 6 | 6 | 6 | 6 | 135 |
| TOTAL | 124 | 34 | 59 | 47 | 36 | 3 |

**Table 4 : Effect on preliminary allocation of an exceptional voting pattern in Transvaal.**

It is clear that party A receives one more seat during the allocation of the first 300 seats than the number it should receive according to the ideal allocation of the full 400 seats. In this very special case the proposed method of compensation will fail.

Example 3

If the voting percentage in Transvaal was 39% and all the other modifications in Transvaal as in Example 2, Table 4 will be left unchanged, but the ideal allocation of the $h = 400$ seats changes to

$$g = (124, 54, 94, 73, 55).$$

In this case party A receives in die preliminary allocation of the first 300 seats exactly the number of seats to which it is entitled in the ideal allocation, in other words A will receive no compensating seats.

## 6. SPECIAL CASES

The Jefferson method does not necessarily yield a unique allocation. For example, if two parties with exactly the same total votes compete for three seats, one of the parties will receive one seat and the other two . The choice of the party to receive two seats is arbitrary. The formulas in Section 3 make provision for this kind of situation.

The example in Table 5 of a region in which eight parties A, B, C, D, E, F, G and H, compete for 15 seats , also illustrates this aspect.

| **Votes cast (in millions)** | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| A | B | C | D | E | F | G | H | TOTAL |
| 0,4 | 0,2 | 0,03 | 0,25 | 0,03 | 0,03 | 0,03 | 0,03 | 1,00 |

**Table 5 : A special situation**

Here

$$\lambda^* = \frac{10^6}{15} = 66666\tfrac{2}{3}.$$

With the choice $\lambda = 50\,001$ the allocation to the parties (in the order A, B, C, D, E, F, G, H) is

$$(7, 3, 0, 4, 0, 0, 0, 0),$$

which means that less than 15 seats are allocated.

With the choice $\lambda = 49\,999$ the allocation is

$$(8, 4, 0, 5, 0, 0, 0, 0),$$

which means that more than 15 seats are allocated.

With the choice $\lambda = 50\,000$ the calculation for the respective parties is as follows (in the notation of Sections 2 and 3):

$$\begin{array}{ll} A: & \lfloor\lfloor 400000/50000 \rfloor\rfloor = 8 \text{ or } 7, \\ B: & \lfloor\lfloor 200000/50000 \rfloor\rfloor = 4 \text{ or } 3, \\ D: & \lfloor\lfloor 250000/50000 \rfloor\rfloor = 5 \text{ or } 4, \\ C, E, F, G, H: & \lfloor\lfloor 30000/500000 \rfloor\rfloor = 0. \end{array}$$

For each of the parties A, B and D two seat allocations can be chosen. This yields three possible valid allocations of the 15 seats, depending upon which party receives the most favourable choice, namely

$$(8, 3, 0, 4, 0, 0, 0, 0)$$

or

$$(7, 4, 0, 4, 0, 0, 0, 0)$$

or

$$(7, 3, 0, 5, 0, 0, 0, 0).$$

For this example $\lambda = 50\,000$ is the unique choice of $\lambda$ which leads to valid Jefferson allocations. The Droop formula

$$\lambda = \frac{10^6}{16} + 1 = 62\,501$$

again does not yield a valid allocation.

## 7. COMPUTER IMPLEMENTATION OF METHOD

A computer program was written in Turbo Pascal V6 to implement the three-phase Jefferson method. The program simulates elections and makes seat allocations to the various parties according to the Jefferson method. The user of the program has two choices to simulate elections. Firstly the program can generate voting percentages to simulate the elections. Voting percentages per region are randomly generated in a given interval (e.g. 50 – 95%). The percentage of the total votes gained by each party in a specific region is also randomly generated. Secondly these values may be given explicitly by the user, from a data file or via the keyboard. The algorithm is as follows.

Global calculation:

*Write input to screen and output file.*

**Call Jefferson method** to allocate provisional seats to the regions (house size 300).

If the quota ($\lambda$) cannot be obtained - **stop execution**.

*Write results to screen and output file.*

For all regions:

**Call Jefferson method** to allocate provisional seats to parties in the relevant region; if $\lambda$ cannot be obtained - **stop execution**.

*Write results to screen and output file.*

**Call Jefferson method** to allocate seats to parties country-wide (house size 400).

If $\lambda$ cannot be obtained - **stop execution.**

*Write results to screen and output file.*

Test for each party whether the global allocation of seats with house size 400 is greater than the sum of the allocations which the party obtained on a regional basis with a house size of 300.

The **Jefferson method** is carried out in a separate procedure. In this procedure an integer value of $\lambda$ which will lead to a valid Jefferson allocation, is sought iteratively. If such a value of $\lambda$ cannot be found, a suitable message is generated. This situation will seldom occur, but if it should happen the user may complete the allocation with hand calculations. If a suitable value of $\lambda$ can be found, the procedure will make the correct Jefferson allocation.

## 8. TEST RUNS

The question in Section 4.5 was investigated by means of the computer program and a number of examples. Data on regions and numbers of eligible voters were taken as in [5]. The values of the parameters in Section 4.5 were used, with the number of parties m = 10.

The program was run several times with values chosen by experts which represented their views of realistic scenarios. In all these cases the 100 compensating seats were sufficient for the allocations which resulted.

The program was also used for 200 runs in two groups of 100 each with random values. In the first group of 100 runs the voting percentage per region was constrained to the range between 50% and 95%. In 98 of the cases, an integer $\lambda$ was found which led to the desired allocation. In these cases the 100 compensating seats were sufficient, in other words each party's global allocation of seats out of 400, based on votes cast country-wide, was always more than the sum of the party's allocations from 300 seats in the regions. In two cases an integer $\lambda$ could not be found. These two cases were analyzed manually and for one a non-integer $\lambda$ was found for which the allocation could be made. For the other case there were two parties which competed for the last seat in the region in question; one had exactly twice the number of votes of the other. In this case the method of Section 6 had to be used to make an allocation. Also in the last two cases the 100 compensating seats were sufficient. The smallest difference between a party's country-wide allocation of seats out of 400, and the sum of its allocations out of 300 seats in the regions, was 3 seats. In this case the party in question gained an exceptionally low number of votes in the regions with an exceptionally high number of voters.

The second group of 100 runs reflected a more pessimistic view of voting percentages in the regions by restricting the percentages to the range between 25% and 75%. The results were similar to those in the first group. In all the cases all three phases of the method were executed successfully.

## 9. CONCLUSIONS

Our simulations for a wide variety of situations indicate that 100 compensating seats are sufficient to yield satisfactory final allocations with the proposed method of proportional representation in South Africa.

## 10. ACKNOWLEDGEMENTS

We would like to thank Prof. H.J. Kotzé of the Department of Political Science at the University of Stellenbosch for bringing the problem to our attention and for elucidating many of its aspects.

## REFERENCES

[1] M.L. BALINSKI, H.P. YOUNG, The Jefferson method of apportionment, *SIAM Review*, 20, 278 - 284, 1978.

[2] M.L. BALINSKI, H.P. YOUNG, Criteria for proportional representation, *Operations Research*, 27, 80 - 95, 1979.

[3] M.L. BALINSKI, H.P. YOUNG, *Fair Representation: Meeting the Ideal of One Man, One Vote*, Yale University Press (1982).

[4] J.-L. PETIT, E. TÉROUANNE, A theory of proportional representation, *SIAM Journal on Discrete Mathematics*, 3, 116 - 139, 1990.

[5] PRESIDENT'S COUNCIL OF THE REPUBLIC OF SOUTH AFRICA, *Report of the Committee for Constitutional Affairs on a Proportional Polling System for South Africa in a New Constitutional Dispensation : PR2/1992*, The Government Printer, Cape Town and Pretoria (1992).