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SEEKING OPTIMALITY IN FRUIT PULPING SCHEDULES: A CASE STUDY

GRANT E HUDDLESTONE & JAN H VAN VUUREN
Department of Applied Mathematics
Stellenbosch University
Private Bag X1
7602 Matieland
South Africa

ABSTRACT

The process of scheduling fruit pulping for the production of fruit juices is of great importance in the beverage industry. Decisions have to be made regarding available processing time, the disposal of fruit that will not be pulped before stock loss due to spoilage, the fulfilment of customer demand and an optimal financial position. Sheduling depends on the capacity of the work force, pulping machine limitations and delivery deadlines. However, the situation is often encountered where the plant manager has to decide which fruit batches (usually from stock piles of overwhelming proportions during the harvesting season) are to be pulped in order to minimize losses due to fruit deterioration. Such decisions are usually done manually, based on intuition and experience. A mathematical model is presented here which constructs a pulping strategy while minimising cascading financial losses associated with fruit grade drops within the stock pile. It is shown in particular that a minimisation of fruit losses is not a good criterion for optimality, and that substantial financial gains may be accomplished when minimising financial losses instead of fruit losses, which is currently standard practice at most fruit pulping plants.

1. INTRODUCTION

There has been a great amount of operations research interest in the management of inventories over the past two decades. This is due to the large scale and complex interactions often encountered during decision making processes concerning inventory systems (see, for example, [2], [3], [10], [11], [13], [15], [17], [18], [20], [24], [25], [26], [27], [29], [30], [31], [32] and [33]). The effects of item perishability cannot be disregarded in many inventory systems, such as in the food & beverages industry as well as in the pharmaceutical sector, where an exclusion of perishability in the analysis leads to an inaccurate mathematical model of the operation of an inventory. Hence the literature on inventory control may be divided into two classes: that of deteriorating item inventories and that of non–deteriorating item inventories. However, within both of these classes researchers have mainly focussed on the situation where some (deterministic or stochastic) demand has to be met and where over–demand is backlogged when insufficient stock or stock intake occurs. Within the class of deteriorating item inventories the goal is then usually to determine an optimal management strategy (in the sense that the total cost of inventory upkeep is minimised) in which inventory stock is used or issued (to meet demand) before items expire (see, for example, [1], [8], [19] and [12] where deterministic demand is considered, and [4], [22], [21], [23] and [28] where stochastic demand is considered).

In deteriorating item inventories a distinction is usually made between two kinds of stock deterioration: the situation where stock level purely diminishes due to deterioration (such as the storage of alcohols and radio active substances), and the situation where certain stock items go bad as a result of deterioration (such as photographic film, pharmaceutical drugs, blood and most food items). Of these two possibilities a stock inventory of the latter kind (which we shall henceforth call a *perishable item inventory*) is much more difficult to manage than the former kind (which we shall call a *diminishing item inventory*), because of the fact that specific delivery batches have to be traced (possibly through different grades in the inventory) and typically have to be processed or discarded before sell–by dates. Diminishing item inventories are usually modelled mathematically by assuming that some (possibly constant) fraction of the on–hand stock deteriorates per time unit (see, for example, [5] and [34]). This simplifying approach cannot, in our view, be used effectively when considering perishable item inventories where different grades or classes of items are present. This is due to the fact that this approach sheds no light on the specific age structure of items within the stock pile (older perishable items typically deteriorate faster than younger ones, and items in different deterioration grades often have substantially different characteristics and qualities). The assumption is sometimes made that deteriorating items are received in a "fresh" state into the inventory and have a fixed life time, essentially resulting in a single grade inventory (see [7] and [9]).

Managing the stock pile when pulping fruit for the production of fruit juices falls within the class of perishable item inventories with multiple deterioration grades. Fruit is typically delivered to a pulping plant by farmers during the harvesting season, and the plant has to make decisions about when to pulp how much of each delivery batch (which may arrive in a variety of different grades or conditions), while fruit in storage is continually subject to deterioration from the present grade to lower grades. In this complex inventory control problem intuitive pulping schedules are questionable due to the sheer size of typical fruit stock piles, and even the formulation of optimality criteria for policies are problematic and vague. Production managers may therefore feel the need for an objective, quantitative analysis of the pulping process to form the basis for decision making.

The problem encountered when scheduling the fruit pulping process is essentially the converse of that usually found in the literature. The problem in most inventory systems is normally to *control* the intake of stock in such a way as to minimise total inventory costs, while satisfying unknown (possibly stochastic) and uncontrollable demand. As we shall see later a fruit pulping plant typically *cannot* control fruit intake effectively (since it is usually a farmers' cooperative and the lowest component in a hierarchy of potential fruit markets). Because of the facts that (i) juice concentrations may be stored for relatively long periods in sealed containers and (ii) demand for juice concentrates are rather intensive (these concentrates are used as bases in which to mix all possible fruit juice flavours), the management of the fruit pulping process is different to standard inventory problems in that an optimal pulping strategy is sought *given* some stock pile profile in inventory. Of course fruit has to be pulped in such a way as to try and satisfy immediate demand, but any juice concentrates not required immediately after production are typically sold by the end of the pulping season or soon thereafter. Hence it is possible to sidestep a stochastic approach to the problem and still obtain a relatively reliable and realistic understanding of the process at hand.

The objectives of this paper are:

- to determine whether an *elementary mathematical treatment* of the pulp scheduling process is possible and feasible in practical terms,
- if so, to establish a *conceptual mathematical model* for drawing up an optimal (in some sense) pulping schedule which may serve as a basis for possible further sophisticated models, and
- using this conceptual model to *evaluate current scheduling practice*, and suggesting possible improvements.

The above mentioned three objectives are pursued in this paper in the following manner. In §2 the process of receiving, grading, storing and pulping fruit, as well as the process of drawing up a typical pulping schedule (as it is currently done at pulping plants) is described in some detail. As is often the case when methods of operations research are employed to solve practical problems in industry, it is necessary to streamline organisation and certain administrative issues at most South African pulping plants before a mathematical model of any kind can be implemented. This is briefly discussed in §3 before developing a system of notation and indices in §4 to facilitate an easy description and interpretaion of variables and parameters in the decision making process at hand. A conceptual mathematical model is derived in §5, paying special attention to constraints applying to the pulping process and to the formulation of a suitable criterion for optimality. The practicality and validity of the underlying model assumptions are also discussed in this section. The general model is summarised in §6, while the complexity of the model solution procedure is addressed in §7. Two numerical examples of pulping schedules are presented and the implications of these examples are discussed in §8. Our conclusions, as well as possibilities for further improvement and generalisation of the mathematical model, follow in §9.

2. DESCRIPTION OF THE FRUIT PULPING PROCESS

Pulping plants usually sign agreements with a number of local fruit farmers whereby agreeing to accept *almost all* fruit that is delivered to the plant so that no shortage in raw material occurs during the South African pulping season (which runs from approximately January to October annually in the Western Cape). Such agreements are valuable to farmers because they constitute a guarantee that there will be a stable market for the fruit crop even if the quality of their produce falls short of the requirements for export and local fresh fruit markets. Hence, from the pulping plant's point of view, there is no control over the intake of raw material and once the height of the harvesting season is reached, the amount of fruit on site is typically of such a magnitude that the effect of fruit loss due to spoilage becomes of primary importance in decisions regarding a pulping schedule. Each grade lifetime depends on various factors such as the kind of fruit and the storage conditions. It is also known that contact with fruit in advanced states of spoilage accelerates the rate of fruit deterioration, as does rain on outside fruit storage spaces. The grade lifetimes can be increased dramatically by refrigerated storage, but this is expensive. The price per fruit crate paid by the plant to the farmer of course depends on the grade of fruit: this price drops with the delivery grade.

Fruit is normally delivered to a pulping plant by farmers in large wooden bins or crates (each holding roughly one cubic metre of fruit). Since farmers are usually not turned away it is clear that all qualities of fruit are received and that some batches may be spoilt in a matter of days (when stored on outside storage areas), while others might last for weeks.

At the height of the season fruit on site at a pulping plant typically reaches overwhelming proportions and hence fruit marked for pulping during upcoming shifts are continually being moved closer to the plant from storage areas. Fruit is normally pulped around the clock during high season, each day typically consisting of a few pulping shifts, each shift in turn followed by a lull during which maintenance and cleaning are done.

The first step in the pulping process is the washing of fruit. Once a fruit batch leaves the washing bins it is put into bins containing an enzyme mixture which weakens the fruit cell walls, thereby allowing juice to be easily extracted by the pulping machines. A pulping machine typically consists of two fabric belts between which the softened fruit is pressed to extract juice. The resulting pulp residue is mixed with water and fed into the

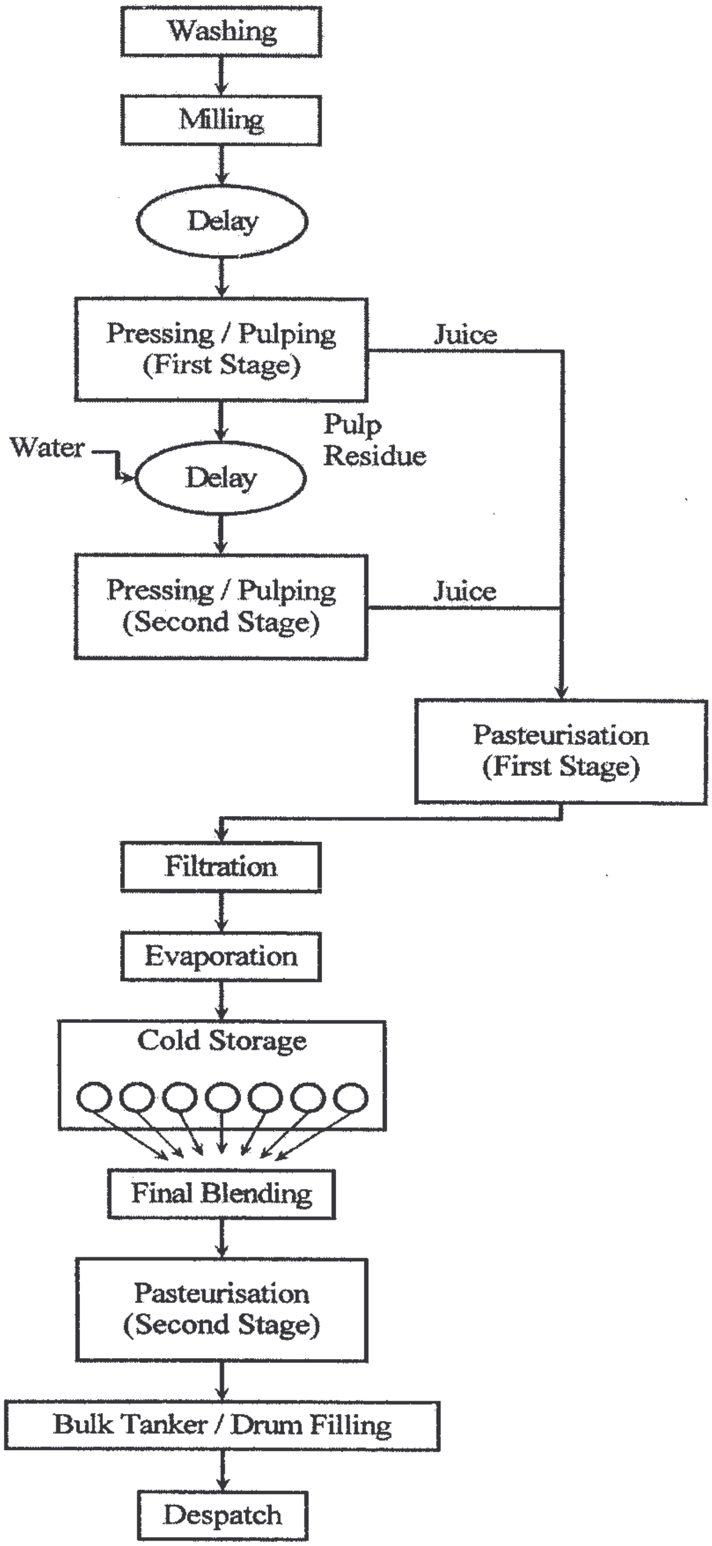


Figure 1: Flow chart of the fruit pulping process at a typical pulping plant.

pulping machines a second time to ensure maximum juice yield. The juice then passes to an evaporator in which most of the water is steamed off, leaving a thick syrup–like concentrate. Each fruit grade yields a juice concentrate with different characteristics as pertaining to sugar and acid contents as well as liquid density, and these concentrates are finally mixed in required proportions so as to meet specific customer demands. The concentrate mixtures are then shipped to customers both locally and internationally. These different stages of the pulping process are illustrated graphically in Figure 1.

3. PROPOSED RE–ORGANISATION AND ADMINISTRATIVE ISSUES

As is often the case with companies in developing countries when an operations research project is initiated, a limited amount of inhouse re–organisation will typically be necessary before any mathematical model for finding optimal schedules can be implemented effectively. The following requirements regarding organisation and administration are proposed to pave the way for a mathematical approach to the decision making process.

3.1 Cascading fruit grade structure. The grading of fruit is a complex issue, depending on various factors, such as sugar, acid and water contents as well as on fruit firmness (amongst others). At present the condition of a fruit batch is typically determined at delivery to the plant by taking a sample of fruit randomly from crates, inspecting the sample and subjecting it to a pressure test. The condition of the batch is determined by considering the number of fruit below the acceptable standard, and by regarding this quantity as a proportion of the sample size. If this proportion is low, the fruit is regarded as being good quality, while if this proportion is high, the batch is branded as poor quality. The price paid by the pulping plant to the farmer for the specific batch depends on the proportion of bad fruit.

It is known that the so called juice yield curve per unit mass of fruit has the general shape depicted in Figure 2. The amount of juice concentrate that can be pulped per volume of fruit depends on the firmness of the fruit, which, in turn, depends on the age of the fruit. Hence there is a critical time span (between when fruit is green and when it is at an advanced stage of deterioration) during which fruit should ideally be pulped.

For this reason it is assumed that fruit can be graded into a discrete number of classes which form a linear cascade, and that each fruit batch starts dropping or cascading through the grade structure immediately after delivery until reaching a final or lowest grade after which it is discarded. This grade partition of the total fruit lifetime may, for example, depend upon the juice yield curve, as shown in Figure 2. It is assumed that an expected lifetime can be allocated to fruit in each grade, so that if the delivery grade of a batch is known, the grade can be determined at any subsequent time.

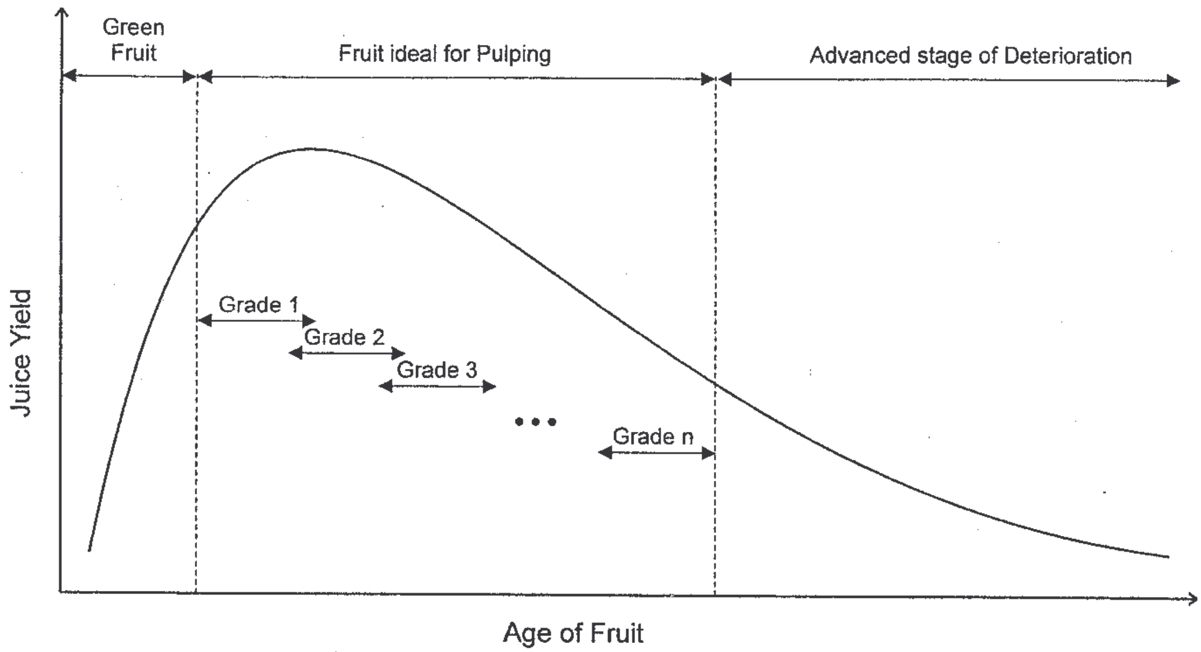


Figure 2: Graphical representation of the juice yield (typically in litres of juice concentrate per ton of pulped fruit) as a function of fruit age. The cascading grade structure of fruit may depend upon the yield per ton of fruit (amongst other things).

3.2 Trapeziodal window arrangement of storage facilities. Fruit awaiting pulping is normally stored in refrigerated store rooms or on outside storage spaces within the wooden crates in which it arrived at the pulping plant. These crates are typically stacked in multiple rows (which may typically be of the order of hundreds of metres long and several metres high) and moved around with fork lifters. Openings or gaps in these fruit stacks usually appear relatively randomly, especially if inventories are managed manually or intuitively. The reason for this is twofold:

- Fruit crates are selected in a heuristic manner for pulping (usually by a production manager, based on his or her experience, in such a way as to try and minimise fruit losses due to deterioration).
- Fruit in different crates are typically in different stages of deterioration (depending on the delivery date and grade for the specific batch).

New batch arrivals are then typically stored in these gaps, resulting in a rather complicated array of stored fruit. To worsen matters, fruit marked for pulping is continually being moved closer to the pulping machines. Hence even the most experienced production manager may find it extremely difficult to keep track of different stages of deterioration of large stock piles, and fruit is inevitably lost due to ineffective inventory management.

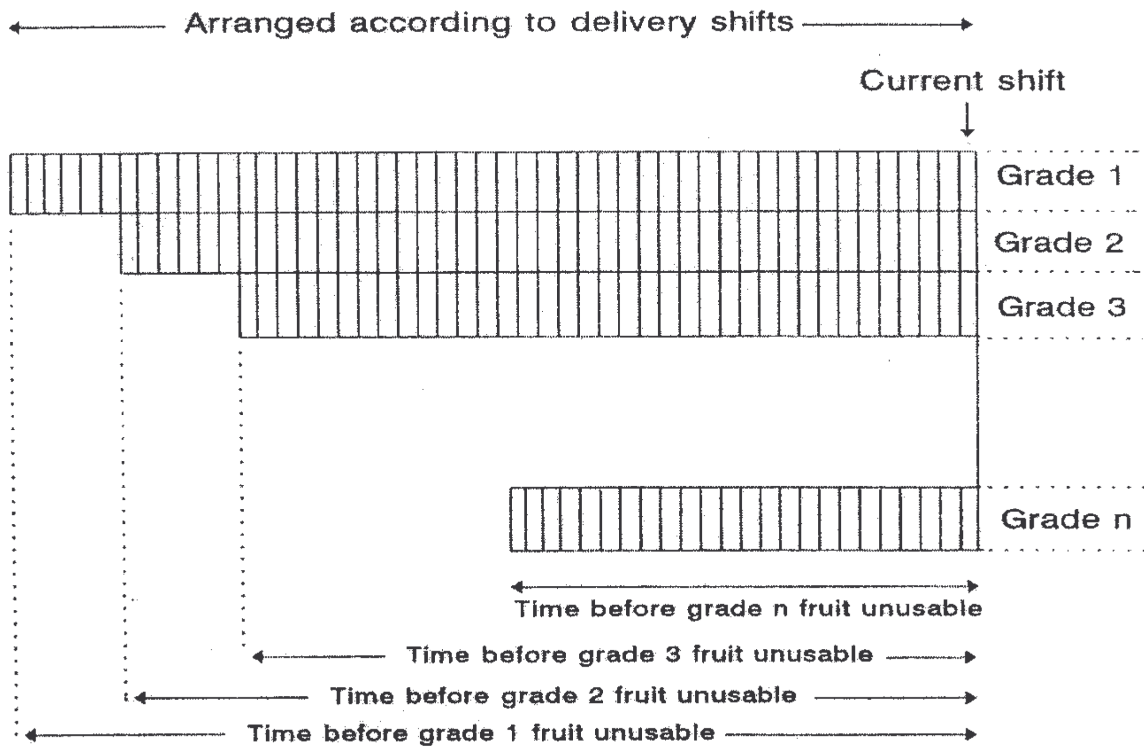


Figure 3: Proposed arrangement of storage facilities at the pulping plant

There is clearly much to be gained from a more orderly arrangement of storage facilities. The arrangement illustrated in Figure 3 is proposed in order to pave the way for the implementation of a mathematical approach to decision making regarding the pulping process. Different rows should ideally be maintained according to the delivery grades of fruit batches. Batches should be stored within rows according to their delivery dates, and these delivery dates should be clearly marked. This results in a trapeziodal–window storage array with the number of rows equal to the number of possible fruit grades in the inventory structure. In such an arrangement it is not necessary to move fruit if the pulping machines are located at a central place, since the technique of *wrapping* can be used to prevent gradual shifts of the storage space due to fruit deterioration out of the usable time span window. In the proposed storage arrangement it would be easy to locate any specific delivery batch marked for pulping by a mathematical model of the decision making process.

3.3 Computerisation of inventory management. Keeping track of the quality and quantity of on–hand stock is a tedious and time consuming task – especially when the process is not computerised, as is the case at many pulping plants. Any mathematical model of the decision making process will require information regarding the unused amounts of fruit in the various delivery batches on a continuous basis. Therefore stock pile information will have to be updated at the end of each pulping shift or cycle to ensure true optimality of the ensuing mathematical model solution. Hence there is clearly a need to computerise the intake and grading of fruit batches in such a manner that information can be electronically updated by fork lift operators or pulping personnel. This may be accomplished by using hand computer units (with built–in bar code detectors) which are compatible with the central computer(s) used for managing the stock inventory. In this way an operator or manager can move around the plant storage site freely, while recording the relevant update information, and later link the hand unit to the central computer(s) to download the necessary updates.

There are various advantages to such a computerisation of the stock keeping process:

- With a fully computerised stock taking facility a clearer picture of the exact nature and composition of the fruit stock pile can be obtained instantly, and such a facility can be used conveniently to compile a variety of deterioration reports on a continuous basis.
- The process of fruit intake, grading, as well as subsequent storage and handling of fruit batches can be affected more efficiently by implementing a computerised stock taking facility. In such a way human error by misplacing or losing track of or forgetting about fruit batches can be minimised.
- By doing computerised stock taking on a continuous basis it would be easier to administrate new orders in the sense that it would be easier to decide on pricing issues and realistic order deadlines.
- An accounting package could ideally be linked to a computerised stock taking facility to do the relevant bookkeeping in a cost effective manner.

4. NOTATION & MODEL ASSUMPTIONS

Let us now turn to the mathematical description of the fruit pulping process. Assume that the pulping season is divided into P pulping cycles, each in turn consisting of r^p pulping shifts $(p = 1, \ldots, P)$. Number the shifts in the pulping season consecutively from 1 to D: this numbering will be referred to as the *global numbering* of shifts within the season. It will also prove convenient to number shifts locally within each pulping cycle. Denote the starting shift of the p–th pulping cycle by d^p $(p = 1, \ldots, P)$. Global shift d^p will therefore be referred to as the first local shift of the p–th pulping cycle, while $d^p + r^p - 1$ is the global number of the last or r^p–th local shift of the cycle. These numberings of shifts are illustrated schematically in Table 1.

As a point of departure we assume that the mission of the pulping plant is to satisfy orders on or before certain deadlines while minimising (fruit and/or financial) losses or maximising profit. Therefore suppose that only one order is processed per pulping cycle and set the start of pulping cycle p just after the final shift of cycle $p-1$ if the p–th

Global shift →	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47
Local shift →	5	6	7		1	2	3	4			1	2	3	4	5	
Pulping cycle →	Cycle $p-1$				Cycle p						Cycle $p+1$					

Table 1: Global versus local numbering of pulping shifts. Here $d^{p-1} = 28$ & $r^{p-1} = 7$; $d^p = 36$ & $r^p = 4$ and $d^{p+1} = 42$ & $r^{p+1} = 5$.

order is received before the end of cycle $p-1$. Otherwise, if this order is received *after* the end of cycle $p-1$, set the start of cycle p on the shift directly following the one in which the order was received. The end of cycle p is defined by the due date for order p: hence this due date is assumed to be feasible regarding machine limitations and workforce restrictions. Since no demand other than specific orders are to be considered, since orders are typically received constantly throughout the season, and since the intake of fruit is an unpredictable dynamic process, a dynamic mathematical model of the scheduling process is desirable – a model which draws up a pulping schedule for the whole pulping season will not do. Therefore each cycle has to be modelled consecutively and separately from the previous ones. Hence any resulting optimal pulping schedule will be optimal in the sense of a one–cycle time frame (where cycle lengths are prompted by individual orders), and will not be optimal in a global (seasonal) sense.

Each fruit juice concentrate is produced by pulping only a single fruit grade and therefore characteristics (such as sugar and acid contents) of each concentrate correspond to the associated fruit grade[1]. Suppose there are n different fruit grades (and hence also n different juice concentrates). However, an order may consist of a number of juice products; each product being a mixture of juice concentrates according to some recipe. Suppose there are m possible different products and that the mixing recipe of the i–th product is contained in the formulation vector $\underline{f}^i = [c_1^i, \ldots, c_n^i]^T$, where the entry c_j^i denotes the amount [number of tons] of pulped grade j fruit necessary to make up one litre of product i ($i = 1, \ldots, m$, $j = 1, \ldots, n$). The $n \times m$ formulation matrix

$$\mathbf{F} = \left[\underline{f}^1, \cdots, \underline{f}^m\right] = \begin{bmatrix} c_1^1 & \cdots & c_1^m \\ \vdots & & \vdots \\ c_n^1 & \cdots & c_n^m \end{bmatrix}$$

therefore contains all the relevant mixing information to produce the juice mixtures as ordered and specified by clients. Suppose that b_i^p litres of juice mixture i is necessary to fill order p (the order being processed during cycle p) by the deadline $d^p + r^p$ (which means that the order should be completed by the end of global shift $d^p + r^p - 1$), then

[1]In practice fruit from different fruit batches may sometimes be mixed before pulping in order to be able to use fruit from very low grades by altering their characteristics after mixture with fruit from higher grades, but this is ignored in our mathematical treatment of the problem. We return to this issue in §9.

the order vector $\underline{b}^p = [b_1^p, \cdots, b_m^p]^T$ fully specifies order p, while the $m \times p$ order matrix

$$\mathbf{B} = \left[\underline{b}^1, \cdots, \underline{b}^p\right] = \begin{bmatrix} b_1^1 & \cdots & b_1^p \\ \vdots & & \vdots \\ b_m^1 & \cdots & b_m^p \end{bmatrix}$$

contains all past and current ordering information. As new orders are received the order matrix is extended by adding column vectors.

Denote the lifetime (due to deterioration) of fruit in grade j by T^j shifts ($j = 1, \ldots, n$), so that the lifetime vector $\underline{T} = [T^1, \cdots, T^n, \infty]^T$ enables us to determine in which grade any delivery batch will be at any future date if only the delivery date and delivery grade are known. The last infinite entry in the lifetime vector signifies that once fruit is lost it remains lost. Note that in assuming a single lifetime for each fruit grade we have in effect ignored different storage conditions such as refrigeration versus outside storage spaces. Suppose further that the pulping plant is prepared to pay y^j Rands per ton of grade j fruit received, and capture these grade prices in the single price vector $\underline{y} = [y^1, \cdots, y^n, 0]^T$, where the last zero entry indicates that lost fruit is worthless.

In order to draw up a pulping schedule it will be necessary to keep track of on hand stock. Therefore let $B_j^k(l)$ denote the amount [number of tons] of unpulped fruit in the inventory *left over* at the end of (global) shift k that was received in grade j during (global) shift l (this amount is continually updated as fruit is pulped from the batch). This notation will enable us to follow specific delivery batches through the inventory in time until they are either entirely processed or else eventually discarded. A batch of fruit that enters the inventory during (global) shift l in grade j is immediately labelled $B_j^l(l)$ and this variable is assigned as value the number of tons received. A batch cannot undergo partial or complete processing during the production cycle in which it was received, because an order that was placed for delivery at the end of any pulping cycle can only be filled by fruit that was already recorded in the inventory at the start of that cycle (the mathematical model is to be implemented on a cycle to cycle basis). Whatever the case may be, a fruit batch has to be in the inventory at least one shift before it can be processed. However, this model "complication" allows time for the administration of stock intake before making stock officially available in the inventory. Every subsequent day after delivery a fruit batch may or may not undergo (partial) processing. This, in turn, means that the initial batch size may either remain the same or reduce as time passes and hence $B_j^l(l) \geq B_j^{l+k}(l)$ for all $k = 1, \ldots, O_n^j$, where the duration $O_\beta^\alpha := T^\alpha + T^{\alpha+1} + \ldots + T^\beta$ denotes the time span [in shifts] necessary for fruit to deteriorate from grade α past grade β ($1 \leq \alpha \leq \beta \leq n$). Table 2 illustrates the cascading of a fruit batch delivered in grade 2 during global shift l through a four grade structured inventory with grade lifetimes defined by $\underline{T} = [4, 3, 2, 1, \infty]^T$.

Global shift →	···	l	$l+1$	$l+2$	$l+3$	$l+4$	$l+5$	$l+6$
Fruit grade 1								
Fruit grade 2		$B_2^l(l)$ →	$B_2^{l+1}(l)$ →	$B_2^{l+2}(l)$				
Fruit grade 3				↪	$B_2^{l+3}(l)$ →	$B_2^{l+4}(l)$		
Fruit grade 4						↪	$B_2^{l+5}(l)$	
Lost fruit							↪	$B_2^{l+6}(l)$

Table 2: A fruit batch cascading through a four grade structured inventory with grade lifetimes $T^1 = 4$, $T^2 = 3$, $T^3 = 2$ and $T^4 = 1$ shifts.

Finally, we define the decision variables $S_j^{p,k}(l)$ as the amount [number of tons] of fruit, from the batch received during (global) shift l in grade j, that should be pulped during (local) shift k of pulping cycle p.

Consider, as an example, a four grade structured inventory with grade lifetimes $T^1 = 4$, $T^2 = 3$, $T^3 = 3$ & $T^4 = 2$ shifts, and suppose that pulping cycle p has a duration of 6 shifts and starts on global shift 50. In this case the pulping schedule will take the form of Table 3 due to the cascading effect of stock through the inventory.

The batch $B_1^{49}(43)$ will be traced through the inventory by means of Table 3 to illustrate the cascading effect of fruit on a pulping schedule. At the start of local shift 1 of the production cycle (global shift 50) the left overs from the batch that was received during global shift 43 in a grade 1 condition has already deteriorated to grade 3. If the whole batch residue is not pulped during or before local shift 3 (global shift 52) then what remains will fall to grade 4 at the end of that shift. Similarly, if the batch residue is not entirely processed by the end of local shift 5 (global shift 54), then what remains at the end of that shift will be lost to spoilage.

It will be our objective in the following section to set up a mathematical model which specifies how much of each fruit grade should ideally be pulped during each of the r^p local shifts of the p–th pulping cycle, in other words constructing a model which specifies the values of $S_j^{p,k}(l)$ in Table 4 in some optimal sense.

5. DERIVATION OF MATHEMATICAL MODEL

The process of finding an optimal pulping schedule will be subject to certain restrictions or constraints, and our mathematical model will take the form of a linear programming problem. We identify four classes of constraints, which, together with the objective function, will be discussed separately in the following subsections.

5.1 Satisfying the current order. The mission of the pulping staff during pulping cycle p will be to (1) satisfy the order that is due by the end of the cycle, and (2) to use the remaining pulping time to (partially) pulp those fruit batches that would incur the greatest financial losses for the company if they were lost or if they deteriorated to

	Global shift →	49	50	51	52	53	54	55	56
	Local shift →	–	1	2	3	4	5	6	–
	Pulping cycle →	–				p			
Purchase Grade 1	Grade 1	$B_1^{49}(49)$	–	–	–	–	–	–	$B_1^{56}(56)$
		$B_1^{49}(48)$	$S_1^{1}(49)$	–	–	–	–	–	$B_1^{56}(55)$
		$B_1^{49}(47)$	$S_1^{1}(48)$	$S_1^{2}(49)$	–	–	–	–	$B_1^{56}(54)$
		$B_1^{49}(46)$	$S_1^{1}(47)$	$S_1^{2}(48)$	$S_1^{3}(49)$	–	–	–	$B_1^{56}(53)$
	Grade 2	$B_1^{49}(45)$	$S_1^{1}(46)$	$S_1^{2}(47)$	$S_1^{3}(48)$	$S_1^{4}(49)$	–	–	$B_1^{56}(52)$
		$B_1^{49}(44)$	$S_1^{1}(45)$	$S_1^{2}(46)$	$S_1^{3}(47)$	$S_1^{4}(48)$	$S_1^{5}(49)$	–	$B_1^{56}(51)$
		$B_1^{49}(43)$	$S_1^{1}(44)$	$S_1^{2}(45)$	$S_1^{3}(46)$	$S_1^{4}(47)$	$S_1^{5}(48)$	$S_1^{6}(49)$	$B_1^{56}(50)$
	Grade 3	$B_1^{49}(42)$	$S_1^{1}(43)$	$S_1^{2}(44)$	$S_1^{3}(45)$	$S_1^{4}(46)$	$S_1^{5}(47)$	$S_1^{6}(48)$	$B_1^{56}(49)$
		$B_1^{49}(41)$	$S_1^{1}(42)$	$S_1^{2}(43)$	$S_1^{3}(44)$	$S_1^{4}(45)$	$S_1^{5}(46)$	$S_1^{6}(47)$	$B_1^{56}(48)$
		$B_1^{49}(40)$	$S_1^{1}(41)$	$S_1^{2}(42)$	$S_1^{3}(43)$	$S_1^{4}(44)$	$S_1^{5}(45)$	$S_1^{6}(46)$	$B_1^{56}(47)$
	Grade 4	$B_1^{49}(39)$	$S_1^{1}(40)$	$S_1^{2}(41)$	$S_1^{3}(42)$	$S_1^{4}(43)$	$S_1^{5}(44)$	$S_1^{6}(45)$	$B_1^{56}(46)$
		–	$S_1^{1}(39)$	$S_1^{2}(40)$	$S_1^{3}(41)$	$S_1^{4}(42)$	$S_1^{5}(43)$	$S_1^{6}(44)$	–
Purchase grade 2	Grade 2	$B_2^{49}(49)$	–	–	–	–	–	–	$B_2^{56}(56)$
		$B_2^{49}(48)$	$S_2^{1}(49)$	–	–	–	–	–	$B_2^{56}(55)$
		$B_2^{49}(47)$	$S_2^{1}(48)$	$S_2^{2}(49)$	–	–	–	–	$B_2^{56}(54)$
	Grade 3	$B_2^{49}(46)$	$S_2^{1}(47)$	$S_2^{2}(48)$	$S_2^{3}(49)$	–	–	–	$B_2^{56}(53)$
		$B_2^{49}(45)$	$S_2^{1}(46)$	$S_2^{2}(47)$	$S_2^{3}(48)$	$S_2^{4}(49)$	–	–	$B_2^{56}(52)$
		$B_2^{49}(44)$	$S_2^{1}(45)$	$S_2^{2}(46)$	$S_2^{3}(47)$	$S_2^{4}(48)$	$S_2^{5}(49)$	–	$B_2^{56}(51)$
	Grade 4	$B_2^{49}(43)$	$S_2^{1}(44)$	$S_2^{2}(45)$	$S_2^{3}(46)$	$S_2^{4}(47)$	$S_2^{5}(48)$	$S_2^{6}(49)$	$B_2^{56}(50)$
		–	$S_2^{1}(43)$	$S_2^{2}(44)$	$S_2^{3}(45)$	$S_2^{4}(46)$	$S_2^{5}(47)$	$S_2^{6}(48)$	–
Purchase grade 3	Grade 3	$B_3^{49}(49)$	–	–	–	–	–	–	$B_3^{56}(56)$
		$B_3^{49}(48)$	$S_3^{1}(49)$	–	–	–	–	–	$B_3^{56}(55)$
		$B_3^{49}(47)$	$S_3^{1}(49)$	$S_3^{2}(49)$	–	–	–	–	$B_3^{56}(54)$
	Grade 4	$B_3^{49}(46)$	$S_3^{1}(47)$	$S_3^{2}(48)$	$S_3^{3}(49)$	–	–	–	$B_3^{56}(53)$
		–	$S_3^{1}(46)$	$S_3^{2}(47)$	$S_3^{3}(48)$	$S_3^{4}(49)$	–	–	–
Purchase grade 4	Grade 4	$B_4^{49}(49)$	–	–	–	–	–	–	$B_4^{56}(56)$
		–	$S_4^{1}(49)$	–	–	–	–	–	–

Table 3: An example of a pulping schedule for a four grade structured inventory with lifetime vector $\underline{T} = [4, 3, 3, 2, \infty]^T$, with $d^p = 50$ and $r^p = 6$. Here the p superscripts have been omitted in order to save space.

a lower grade. As already mentioned the amount [number of litres] of the i–th product required for the order due just after the end of the p–th pulping cycle is b_i^p, while one litre of the i–th product requires the pulping of c_j^i tons of grade j fruit. Therefore $b_i^p c_j^i$ tons of grade j fruit is necessary to produce the required amount of product i for order p. Hence, for a pulping cycle of the dimensions of Table 3, the sum of the pulping variables associated with grade 2 fruit should not be less than $\sum_{i=1}^m b_i^p c_2^i$, or symbolically,

$$\begin{array}{ll}
 & S_1^{p,1}(46) + S_1^{p,2}(47) + S_1^{p,3}(48) + S_1^{p,4}(49) \\
+ & S_1^{p,1}(45) + S_1^{p,2}(46) + S_1^{p,3}(47) + S_1^{p,4}(48) + S_1^{p,5}(49) \\
+ & \underline{S_1^{p,1}(44) + S_1^{p,2}(45) + S_1^{p,3}(46) + S_1^{p,4}(47) + S_1^{p,5}(48) + S_1^{p,6}(49)} \\
+ & S_2^{p,1}(49) \\
+ & S_2^{p,1}(48) + S_2^{p,2}(49) \qquad \geq \sum_{i=1}^m b_i^p c_2^i
\end{array}$$

in order to be able to produce the necessary amounts of all m products contained in the

p–th order. The general form of this constraint is therefore

$$\sum_{k=1}^{\min\{r^p, O_n^j - 1\}} \sum_{l=1}^{O_n^j - 1} S_j^{p,k}(d^p - l) \geq \sum_{i=1}^{m} b_i^p c_j^i, \quad j = 1, \ldots, n. \tag{1}$$

This set of constraints may be discarded during lulls between specific order cycles.

5.2 Utilising machine & workforce capacity. The total pulping amount for each shift is required to be equal to the maximal (constant) pulping rate, L tons per shift (say). It is clear that the total amount of pulped fruit per shift cannot exceed the upper limit L due to machine limitations and workforce capacity. However, by requiring that this upper limit is in fact attained during each shift we ensure that time does not go idly by, but that the pulping machines work at full capacity around the clock (except, of course, when maintenance is done). Hence the model will *first* ensure that at least the amounts of fruit necessary to fill order p is pulped during cycle p, and *then* use any spare time to pulp fruit in such a way that losses due to fruit spoilage are minimised in some sense. If Table 3 is consulted for local shift 2, then the associated constraint is

$$\begin{array}{rcl} S_1^{p,2}(49) + S_1^{p,2}(48) + S_1^{p,2}(47) + S_1^{p,2}(46) & & \\ + S_1^{p,2}(45) + S_1^{p,2}(44) + S_1^{p,2}(43) + S_1^{p,2}(42) + S_1^{p,2}(41) + S_1^{p,2}(40) & & \\ \hline + S_2^{p,2}(49) + S_2^{p,2}(48) + S_2^{p,2}(47) + S_2^{p,2}(46) + S_2^{p,2}(45) + S_2^{p,2}(44) & & \\ \hline + S_3^{p,2}(49) + S_3^{p,2}(48) + S_3^{p,2}(47) & = & L, \end{array}$$

or, in a slightly generalized form,

$$\begin{array}{rcl} S_1^{p,k}(d^p - 1) + S_1^{p,k}(d^p - 2) + \cdots + S_1^{p,k}(d^p - (O_n^1 - k)) & & \\ + S_2^{p,k}(d^p - 1) + S_2^{p,k}(d^p - 2) + \cdots + S_2^{p,k}(d^p - (O_n^2 - k)) & & \\ + S_3^{p,k}(d^p - 1) + S_3^{p,k}(d^p - 2) + \cdots + S_3^{p,k}(d^p - (O_n^3 - k)) & = & L, \end{array}$$

where $n = 4$, $k = 2$, $d^p = 50$, $O_n^1 = 12$, $O_n^2 = 8$ and $O_n^3 = 5$, since $T = [4, 3, 3, 2, \infty]^T$. Hence the general form of this constraint is

$$\sum_{j=1}^{n} \sum_{l=1}^{O_n^j - k} S_j^{p,k}(d^p - l) = L, \quad k = 1, \ldots, r^p. \tag{2}$$

5.3 Observing inventory limitations. No more fruit can be pulped from any delivery batch during any pulping shift than was available in the inventory at the start of that cycle less the amounts of fruit previously pulped within the cycle. For example, the amount of fruit pulped for variable $S_1^{p,4}(47)$ in Table 3 should not exceed $B_1^{49}(47) - S_1^{p,1}(47) - S_1^{p,2}(47) - S_1^{p,3}(47)$. Therefore

$$S_1^{p,1}(47) + S_1^{p,2}(47) + S_1^{p,3}(47) + S_1^{p,4}(47) \leq B_1^{49}(47),$$

or, in a slightly generalized form,

$$S_j^{p,1}(d^p - l) + S_j^{p,2}(d^p - l) + S_j^{p,3}(d^p - l) + S_j^{p,4}(d^p - l) \leq B_j^{d^p - 1}(d^p - l),$$

where $j = 1$ and $l = 3$. Hence the general form of this constraint is

$$\sum_{k=1}^{\min\{r^p, O_n^j - 1\}} S_j^{p,k}(d^p - l) \leq B_j^{d^p-1}(d^p - l), \quad \left\{ \begin{array}{l} l = 1, \ldots, O_n^j - 1 \\ j = 1, \ldots, n. \end{array} \right. \tag{3}$$

5.4 Non–negativity of schedule entries. Of course we finally have to require that $S_j^{p,k}(l)$ is non–negative for all $j, l = 1, \ldots, n$ and $k = 1, \ldots, r^p$ since negative pulping quantities do not make sense.

5.5 The objective function. In order to complete the model formulation we need to specify a criterion for optimality in which context an optimal pulping schedule is sought. As was explained in §2, pulping plants are typically farmers' cooperatives and are therefore not in a position where the intake of fruit can be controlled. For this reason a maximisation of profit as model objective is not an ideal platform for an optimal scheduling of the decision process. If farmers could be turned away or if there were often vacant times between orders, an objective function that maximises profit would be meaningful: the mathematical model would then be required to determine which fruit grades (and how much of each) should be taken in so as to maximise financial gain. Such an approach would of course depend on (stochastic) product demand as well as selling prices. However, as stated before, it is *not* plant policy to turn away fruit suppliers and machine time is also typically *not* abundant. The only other way in which financial gain may be accomplished is by minimising losses (i.e. fruit losses or financial losses, or both).

The pulping plant has unavoidable losses due to the need to discard fruit which is not useful during the span of the current set of orders. However, if grade j fruit is bought at Ry^j per ton and is allowed to deteriorate to grade $j+1$ for which the purchase price is Ry^{j+1} per ton (with $y^j > y^{j+1}$), then the company has spent at least R$(y^j - y^{j+1})$ per ton too much for that specific fruit batch, which will consequently be pulped in grade $j+1$ or lower. It is therefore clear that the monetary loss associated with incorrect scheduling can have a marked effect on the financial position of the company. We choose to minimise this cascading set of monetary losses, since this is more general than merely minimising fruit losses in the lowest grade. It may, for example, be possible to obtain a smaller financial loss by allowing fruit in low grades to be lost to spoilage, while preventing (expensive) fruit in higher grades to deteriorate to lower grades.

Consider, as an example, losses incurred only by fruit that deteriorates from grade 3 to grade 4 during (local) shift 5 of the pulping cycle in Table 3. It is clear (by following diagonals in the table) that such losses occur only if fruit from batches $B_1^{49}(45)$ and $B_2^{49}(49)$ is left over at the end of (local) shift 5. Grade 3 to grade 4 deterioration by fruit from other batches occur either before or after local shift 5 (if at all). Hence the amount of fruit deteriorating from grade 3 to grade 4 at the end of local shift 5 totals

$$B_1^{49}(45) - S_1^{p,1}(45) - S_1^{p,2}(45) - S_1^{p,3}(45) - S_1^{p,4}(45) - S_1^{p,5}(45) \tag{4}$$

together with

$$B_2^{49}(49) - S_2^{p,1}(49) - S_2^{p,2}(49) - S_2^{p,3}(49) - S_2^{p,4}(49) - S_2^{p,5}(49) \tag{5}$$

tons. In order to minimise the financial loss due to this deterioration we therefore have to

$$\begin{aligned}\text{Minimise}\quad & (y^3 - y^4)[\{B_1^{49}(45) - S_1^{p,1}(45) - S_1^{p,2}(45) - S_1^{p,3}(45) \\ & - S_1^{p,4}(45) - S_1^{p,5}(45)\} + \{B_2^{49}(49) - S_2^{p,1}(49) - S_2^{p,2}(49) \\ & - S_2^{p,3}(49) - S_2^{p,4}(49) - S_2^{p,5}(49)\}],\end{aligned}$$

which is a financially weighted sum of the relevant fruit deterioration amounts. The total loss for the pulping cycle is obtained by summing over all (local) shifts (1 to 6) as well as over all possible grade deteriorations (grade 1 $\rightarrow$ 2, grade 2 $\rightarrow$ 3, grade 3 $\rightarrow$ 4 and grade 4 $\rightarrow$ lost in the case of this four grade structured inventory example).

Due to the cascading effect of deteriorating stock in inventory as illustrated in Table 2 the amount of grade j fruit available for pulping during local shift k of the p–th pulping cycle is given by

$$\sum_{\gamma=1}^{\Gamma}\sum_{l=1}^{T^\gamma}\left[B_\gamma^{d^p-1}(d^p+k+l-O_j^\gamma-1)-\sum_{\kappa=1}^{k-1}S_\gamma^{p,\kappa}(d^p+k+l-O_j^\gamma-1)\right], \tag{6}$$

where Γ is the largest integer from the set $\{1,\ldots,j\}$ such that $O_j^\gamma \geq k+1$. This formula can be obtained by direct back–tracking of the inventory. However, not all the stock in (6) is subject to deterioration at the end of local shift k of the p–th cycle; only those fruit not pulped during the k–th local shift and corresponding to the choice $l = 1$ will deteriorate to grade $j+1$ at the end of local shift k. Therefore only the amount

$$\sum_{\gamma=1}^{\Gamma}\left[B_\gamma^{d^p-1}(d^p+k-O_j^\gamma)-\sum_{\kappa=1}^{k}S_\gamma^{p,\kappa}(d^p+k-O_j^\gamma)\right] \tag{7}$$

will deteriorate from grade j to grade $j+1$ by the end of local shift k. It is clear that the amounts in (4) and (5) may be obtained from (7) with the choice $j = 3$ and $k = 5$, together with $d^p = 50$ and $r^p = 6$ as in Table 3. The financial loss associated with the fruit loss (7) is given by

$$(y^j - y^{j+1})\sum_{\gamma=1}^{\Gamma}\left[B_\gamma^{d^p-1}(d^p+k-O_j^\gamma)-\sum_{\kappa=1}^{k}S_\gamma^{p,\kappa}(d^p+k-O_j^\gamma)\right],$$

and hence the total cascading financial loss for the p–th pulping cycle is given by

$$\sum_{j=1}^{n}(y^j - y^{j+1})\sum_{k=1}^{r^p}\sum_{\gamma=1}^{\Gamma}\left[B_\gamma^{d^p-1}(d^p+k-O_j^\gamma)-\sum_{\kappa=1}^{k}S_\gamma^{p,\kappa}(d^p+k-O_j^\gamma)\right]. \tag{8}$$

Our mathematical model will therefore be required to minimise the total cascading financial loss in (8) during pulping cycle p.

6. THE GENERAL MATHEMATICAL MODEL

The mathematical model for drawing up an optimal pulping schedule is therefore the linear program,

Minimize

$$\sum_{j=1}^{n}(y^j - y^{j+1})\sum_{k=1}^{r^p}\sum_{\gamma=1}^{\Gamma}\left[B_\gamma^{d^p-1}(d^p + k - O_j^\gamma) - \sum_{\kappa=1}^{k} S_\gamma^{p,\kappa}(d^p + k - O_j^\gamma)\right]$$

Subject to :

$$\begin{aligned}
\sum_{k=1}^{\min\{r^p, O_n^j - 1\}} \sum_{l=1}^{O_n^j - k} S_j^{p,k}(d^p - l) &\geq \sum_{i=1}^{m} b_i^p c_j^i, & j = 1, \ldots, n \\
\sum_{j=1}^{n} \sum_{l=1}^{O_n^j - k} S_j^{p,k}(d^p - l) &= L, & k = 1, \ldots, r^p \\
\sum_{k=1}^{\min\{r^p, O_n^j - 1\}} S_j^{p,k}(d^p - l) &\leq B_j^{d^p-1}(d^p - l), & \begin{cases} l = 1, \ldots, O_n^j - 1 \\ j = 1, \ldots, n. \end{cases}
\end{aligned}$$

This model contains $\sum_{j=1}^{n}\sum_{k=1}^{r^p} \max\{0, O_n^j - k\}$ variables and $r^p + \sum_{j=1}^{n} O_n^j$ non–trivial constraints together with the $r^p\sum_{j=1}^{n}(O_n^j - 1)$ trivial constraints

$$S_j^{p,k}(l) \geq 0, \quad \begin{cases} l = 1, \ldots, O_n^j - k \\ j = 1, \ldots, n \\ k = 1, \ldots, r^p. \end{cases}$$

The meaning of the above symbols are as follows:

n:		Number of fruit grades in the inventory
m:		Number of possible products which may be ordered
y^j:	[Rands]	Price per ton of grade j fruit
T^j:	[Shifts]	Lifespan of grade j fruit
r^p:	[Shifts]	Length of pulping cycle p
d^p:		(Global) shift number on which pulping cycle p starts
$S_j^{p,k}(l)$:	[Tons]	Amount of fruit, from the batch received during (global) shift l in grade j, that should be pulped during (local) shift k of pulping cycle p
O_β^α:	[Shifts]	Time span of deterioration from grade α past grade β
b_i^p:	[Litres]	Amount of product i needed to fill the order for pulping cycle p
c_j^i:	[Tons]	Amount of pulped grade j fruit necessary to make up one litre of product i
L:	[Tons]	Capacity per shift of the pulping machines.

7. SOLVING THE MODEL

In view of the large dimensions of the linear program the numerical solution procedure for the mathematical model was implemented in the high level programming language *MatLab 4.2* which is ideal when having to manipulate large vectors and matrices. A program was written to implement the revised simplex method.

As expected, the model complexity increases as the number of shifts in the pulping cycle increases, or as the number of fruit grades in the inventory structure increases, or as the lifetime of fruit within any particular grade increases. However, the complexity increases most dramatically as the number of fruit grades increases. An increase in the number of shifts in the pulping cycle, on the other hand, only affects a relatively moderate increase in complexity, as is illustrated in Table 4 for the special case where all grade lifetimes are equal.

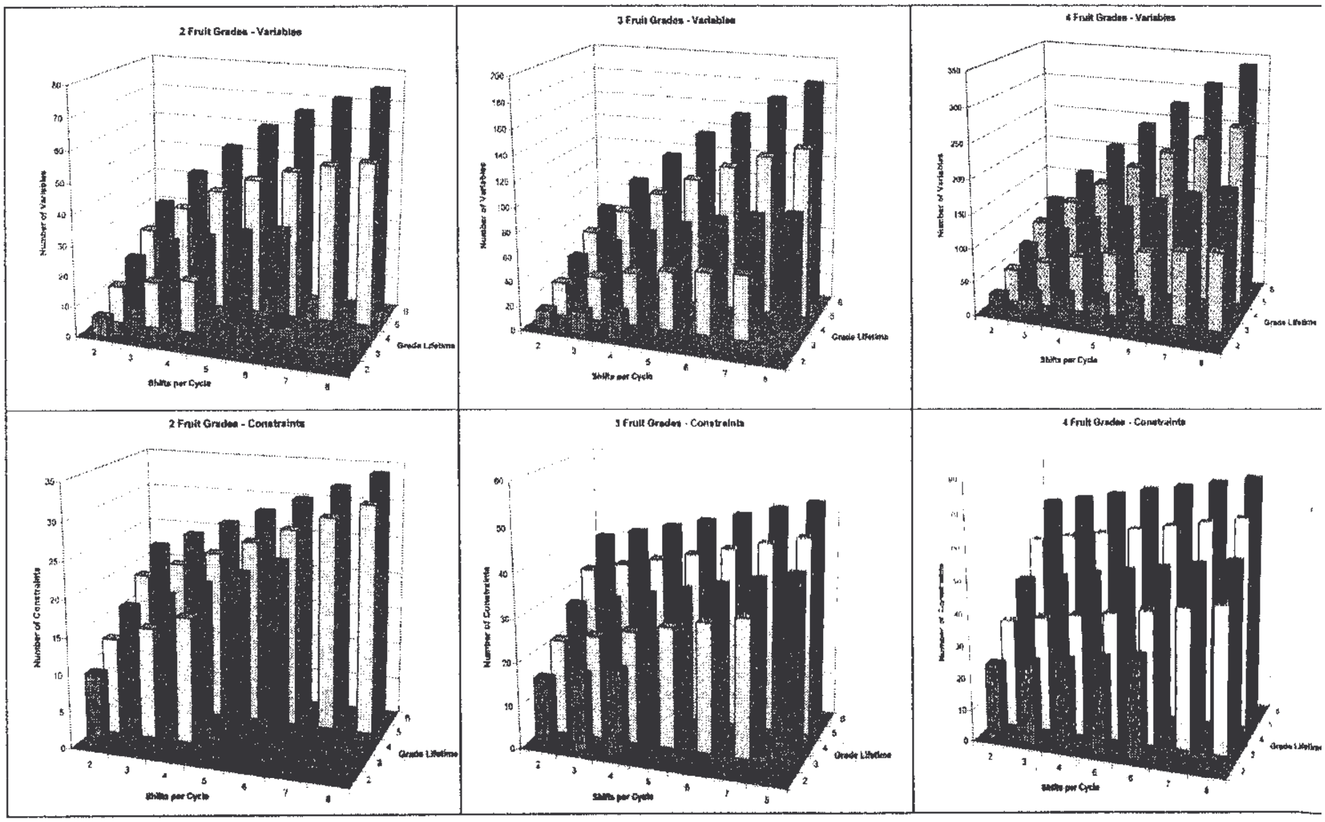
In spite of the large number of floating point operations required to solve the model for a relatively short pulping cycle and a relatively simple grade structured inventory, the solution procedure may still be carried out on a fast personal computer (such as a *Pentium 200 MMX*) in a very reasonable time. It is, in our opinion, therefore not necessary to employ heuristic methods to obtain only "good" pulping schedules because of the size of the model (see, for example, [16]) – an *optimal* solution may be found relatively quickly due to the efficiency of the *Revised Simplex Method* as well as the speed of modern computers. Orders processed by pulping plants may typically require an optimal solution to the decision making process on a weekly basis (maybe resulting in cycles of roughly 7–14 pulping shifts) and for a four or five grade structured inventory (a more complex grade structure would be too difficult to monitor and manage, and would be too finely calibrated to be realistically implemented). A mathematical model of such dimensions could easily be solved on, say, a Friday afternoon or Saturday morning to determine an optimal pulping schedule for the following week.

8. EXAMPLES OF PULPING SCHEDULES

In this section we present two numerical examples of pulping schedules in order to illustrate how an optimal pulping strategy depends on the underlying price structure, as paid by the plant to farmers delivering fruit[2]. Some comments regarding the model complexity are also made in both cases.

Suppose the pulping plant is able to pulp at most 50 tons of fruit per shift and that a six–shift pulping schedule must be drawn up as from global shift 50 onwards for an order of 80 tons, 80 tons, 60 tons and 40 tons of respectively grade 1, 2, 3 and 4 fruit due for delivery during global shift 56. Suppose the grade lifetimes are 4, 3, 3 and 2 shifts for

[2]Due to the sensitive nature of data and decisions regarding the pulping process, all numerical values presented in this section are fictitious.



Shifts per Cycle	Number of Fruit Grades in the Inventory														
	2 Fruit Grades					3 Fruit Grades					4 Fruit Grades				
	Grade Lifetime [shifts]					Grade Lifetime [shifts]					Grade Lifetime [shifts]				
	2	3	4	5	6	2	3	4	5	6	2	3	4	5	6
2	6	12	18	24	30	15	27	39	51	63	28	48	68	88	108
	(10)	(13)	(16)	(19)	(22)	(16)	(22)	(28)	(34)	(40)	(24)	(34)	(44)	(54)	(64)
3		15	25	33	42	19	36	54	72	90	37	66	96	126	156
		(15)	(18)	(21)	(24)	(18)	(24)	(30)	(36)	(42)	(26)	(36)	(46)	(56)	(66)
4		17	28	40	52	21	43	66	90	114	43	81	120	160	202
		(17)	(20)	(23)	(26)	(20)	(26)	(32)	(38)	(44)	(28)	(38)	(48)	(58)	(68)
5			31	45	60		48	76	105	135	47	93	141	190	240
			(22)	(25)	(28)		(28)	(34)	(40)	(46)	(30)	(40)	(50)	(60)	(70)
6			33	49	66		51	84	118	153	49	102	159	217	276
			(24)	(27)	(30)		(30)	(36)	(42)	(48)	(32)	(42)	(52)	(62)	(72)
7				52	71		53	90	129	169		109	174	241	309
				(29)	(32)		(32)	(38)	(44)	(50)		(44)	(54)	(64)	(74)
8				54	75			94	138	183		114	186	262	339
				(31)	(34)			(40)	(46)	(52)		(46)	(56)	(66)	(76)

Table 4: Complexity of the mathematical model. This table contains the number of variables (and between parentheses the number of non–trivial constraints) in the mathematical model as a function of the number of pulping shifts per cycle and the number of fruit grades in the inventory for the special case where all fruit grade lifetimes are equal.

Delivery Grade	Delivery Shift										
	39	40	41	42	43	44	45	46	47	48	49
1	30	0	40	40	50	70	60	80	70	80	80
2	–	–	–	–	20	60	30	40	90	80	100
3	–	–	–	–	–	–	–	10	20	90	90
4	–	–	–	–	–	–	–	–	–	–	50

$O_4^4 - 1 = 1$

$O_4^3 - 1 = 4$

$O_4^2 - 1 = 7$

$O_4^1 - 1 = 11$

Table 5: Distribution of fruit stock pile residue of 1280 (in tons) at the end of global shift 49 for a four grade structured inventory with lifetime vector $T = [4, 3, 3, 2, \infty]^T$. This table is a special case of Table 2 for the numerical examples presented in §8.

grades 1, 2, 3 and 4 respectively and consider an on–hand stock pile of 1280 tons of fruit at the end of global shift 49, structured as in Table 5.

Example 8.1. If the plant pays R250, R210, R130 and R70 per ton of respectively grade 1,2,3 and 4 fruit, an optimal pulping schedule is given in Table 6. The associated financial loss for this pulping schedule during the six pulping shifts amounts to R149 400, while the amount of fruit *lost* (excluding grade deteriorations which did not lead to the discarding of fruit) totals 820 tons. The 460 tons of usable fruit left over at the end of the pulping cycle is contained in the last column of Table 6, labelled global shift 56 (these amounts exclude new deliveries during the course of the cycle). The breakdown of the various grade amounts pulped appear in Table 7. For this example $n = 4$ and $r^p = 6$ so that there are 89 variables and 128 constraints (of which 39 are non–trivial)[3] in the model. Altogether 115 440 724 floating point operations were performed to solve the mathematical model in order to find the schedule presented in Table 6. It took 8.18 seconds to determine the solution on a *Pentium 200 MMX* personal computer.

Note that, since the plant's pulping capacity equals 50 tons per shift, only 300 tons of fruit can be pulped during the six–shift pulping cycle. However, an order of 80 tons, 80 tons, 60 tons and 40 tons of respectively grade 1, 2, 3 and 4 fruit has to be filled by the end of the cycle. This totals 260 tons of fruit that should be pulped in order to fill the order, hence leaving room for pulping an extra 40 tons of fruit in addition to the specific order earmarked for the current pulping cycle. The schedule in Table 6 indicates that all

[3]The equality constraints (ensuring that the machines pulp at full capacity around the clock) are counted twice, since the Revised Simplex Method requires a constraint system of the form $\mathbf{A}\underline{x} \leq \underline{b}$. Therefore any constraint of the form $f(\underline{x}) = L$ is replaced by the constraint pair $f(\underline{x}) \leq L$ and $-f(\underline{x}) \leq -L$.

of the additional 40 tons of pulping capacity should be used to pulp grade 2 fruit. This is due to the relatively large price drop (of R80 per ton) between grades 2 and 3, as shown in Table 7. Since it would be most expensive (per ton of fruit) to let fruit deteriorate from grade 2 to grade 3 the mathematical model automatically selects grade 2 fruit to be pulped during the "free time" of the pulping cycle.

Example 8.2. If, however, the plant pays R250, R210, R160 and R100 per ton of grade 1,2,3 and 4 fruit respectively, a different optimal pulping schedule is found, as shown in Table 8. The associated financial loss for this pulping schedule during the six pulping shifts amounts to R145 584, while the amount of fruit *lost* (excluding grade deteriorations which did not lead to the discarding of fruit) totals 800 tons. The 480 tons of fruit left over at the end of the pulping cycle is contained in the last column of Table 8, labelled global shift 56 (these amounts exclude new deliveries during the course of the cycle). The breakdown of the various grade amounts pulped appear in Table 7. For this example there again are 89 variables and 128 constraints (of which 39 are non–trivial) in the model. Altogether 113 142 448 floating point operations were necessary to solve the mathematical model in order to find the schedule presented in Table 8. It took 7.85 seconds to determine the solution on a *Pentium 200 MMX* personal computer.

Again there is room for pulping an extra 40 tons of fruit in addition to the specific order earmarked for the pulping cycle. The schedule in Table 8 indicates that this time the additional 40 tons of pulping capacity should be used to pulp an extra 8.81 tons of grade 3 fruit, and 31.19 tons of grade 4 fruit. This is due to the relatively large price drops (of respectively R60 and R100 per ton of fruit) between grades 3 & 4 and between grade 4 and lost fruit, as shown in Table 7. Since it would be most expensive (per ton of fruit) to let fruit deteriorate from grade 4 to the lost category, the mathematical model automatically selects a large amount of grade 4 fruit to be pulped during the "free time" of the pulping cycle.

9. DISCUSSION

In this paper a conceptual mathematical model for the computation of optimal fruit pulping schedules was formulated and implemented in a computer program. The model, in the form of a linear programming optimisation problem with a cascading objective function (involving fianancial losses as opposed to fruit losses), showed that a relatively elementary mathematical treatment of the pulp scheduling process is possible and also feasible in practical terms, although the solution process tends to be computationally intensive (even for small structured fruit inventories). However, the model can still be solved within a reasonable time on a fast personal computer due to the efficiency of the Revised Simplex Method.

The mathematical model was also used to critically evaluate current scheduling practice at typical pulping plants, namely an attempt to pulp fruit in such a manner so as to minimise total fruit loss. It was found that this scheduling policy is *not* necessarily optimal from a financial piont of view. In fact, financial optimality is obtained when the most expensive grade drops are avoided or minimised by the pulping schedule. It may therefore be financially more viable to allow large amounts of low grade fruit (for which the purchase price was originally low) to deteriorate to the point of being lost, while preventing high grade fruit (for which the original purchase price was high) to drop to lower grades.

The mathematical modelling approach adopted here may be generalised in two significant ways. First, the assumption of a simple, linearly cascading grading structure within the fruit inventory may be improved. As was pointed out, the condition of fruit (and the subsequent characteristics of juice extracted from it) depends upon a variety of factors such as sugar, acid and water contents as well as on fruit firmness. However, the exact functional dependence is rather complex, and a more realistic (probably non–linear multi–factor) grading structure would be desirable. Mixture of fruit from various batches prior to pulping may also be incorporated into a nonlinear cascading grade structure by allowing the "renewal" of old fruit when mixed with fruit from a high grade. Care should nevertheless be taken that the grading structure does not become so complex that it cannot be implemented and monitored efficiently and on a continual basis from a practical point of view. Secondly, apples and pears are most commonly used to form base juice concentrates (in which to mix other flavours). However, apples and pears cannot be pulped simultaneously on the same pulping line because of the vastly differing characteristics of the juice extracted from these. When there is only one pulping line the decision as to when to pulp apples and when to pulp pears becomes important, since switch–overs are expensive (pulping machines have to be washed after a shift during which apples were pulped if pears are to be pulped during the next shift, and vice versa) and should be avoided as much as possible. It would therefore be desirable to generalise the model so that either optimality is sought via more than one pulping line (as was done in [14] and [6] using a different modelling approach), or else via one pulping line in such a manner that the number of fruit type switch–overs is minimised when pulping.

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Global shift →		49	50	51	52	53	54	55	56
Local shift →		–	1	2	3	4	5	6	Left
Pulping cycle →		–	p						↓ Over
Purchase Grade 1	Grade 1	80	–	–	–	–	–	–	–
		80	0.0000	–	–	–	–	–	–
		70	0.0000	17.0041	–	–	–	–	–
		80	0.0000	32.9959	30.0000	–	–	–	–
	Grade 2	60	0.0000	0.0000	0.0000	0.0000	–	–	–
		70	0.0000	0.0000	0.0000	0.0000	11.0124	–	–
		50	25.0826	0.0000	10.0000	0.0000	27.0041	21.9835	–
	Grade 3	40	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		40	0.0000	0.0000	2.6240	0.0000	0.0000	0.0000	20.0000
		0	0.0000	0.0000	2.4587	25.0826	0.0000	0.0000	70.0000
	Grade 4	30	0.0000	0.0000	0.0000	0.0000	3.1198	0.0000	70.0000
		–	0.0000	0.0000	0.0000	0.0000	2.9545	14.0909	60.0000
	Lost	–	–	30.0000	0.0000	40.0000	40.0000	44.5868	0.0000
Purchase grade 2	Grade 2	100	–	–	–	–	–	–	–
		80	0.0000	–	–	–	–	–	–
		90	24.9174	0.0000	–	–	–	–	–
	Grade 3	40	0.0000	0.0000	0.0000	–	–	–	–
		30	0.0000	0.0000	2.4587	0.0000	–	–	–
		60	0.0000	0.0000	2.4587	24.9174	0.0000	–	–
	Grade 4	20	0.0000	0.0000	0.0000	0.0000	2.9545	0.0000	–
		–	0.0000	0.0000	0.0000	0.0000	2.9545	13.9256	100.000
	Lost	–	–	20.0000	60.0000	30.0000	40.0000	84.5868	10.8264
Purchase grade 3	Grade 3	90	–	–	–	–	–	–	–
		90	0.0000	–	–	–	–	–	–
		20	0.0000	0.0000	–	–	–	–	–
	Grade 4	10	0.0000	0.0000	0.0000	–	–	–	–
		–	0.0000	0.0000	0.0000	0.0000	–	–	–
	Lost	–	–	10.0000	20.0000	90.0000	90.0000	–	–
Purchase grade 4	Grade 4	50	–	–	–	–	–	–	–
		–	0.0000	–	–	–	–	–	–
	Lost	–	–	50.0000	–	–	–	–	–
Total Pulped →		–	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000	–

Table 6: Optimal pulping schedule for Example 8.1 (a four grade structured inventory with lifetime vector $\underline{T} = [4, 3, 3, 2, \infty]^T$, with $d^p = 50$ and $r^p = 6$). The initial fruit stock for the pulping cycle is given in Table 5, while the fruit pricing structure is given in Table 7.

	Example 8.1				Example 8.2			
Grade	Price per Ton [Rands]	Price Drop [Rands]	Amount Pulped [Tons]	Pulped Extra [Tons]	Price per Ton [Rands]	Price Drop [Rands]	Amount Pulped [Tons]	Pulped Extra [Tons]
1	250.00	–	80.00	0.00	250.00	–	80.00	0.00
2	210.00	40.00	120.00	40.00	210.00	40.00	80.00	0.00
3	130.00	80.00	60.00	0.00	160.00	50.00	68.81	8.81
4	70.00	60.00	40.00	0.00	100.00	60.00	71.19	31.19
Lost	0.00	70.00	–	–	0.00	100.00	–	–

Table 7: Breakdown of the required as well as extra amounts of fruit pulped per grade.

Global shift →		49	50	51	52	53	54	55	56
Local shift →		–	1	2	3	4	5	6	Left
Pulping cycle →		–	p						↓ Over
Purchase Grade 1	Grade 1	80	–	–	–	–	–	–	–
		80	0.0000	–	–	–	–	–	–
		70	0.0000	24.3608	–	–	–	–	–
		80	0.0000	24.3608	31.2783	–	–	–	–
	Grade 2	60	0.0000	0.0000	0.0000	0.0000	–	–	–
		70	0.0000	0.0000	0.0000	0.0000	6.2053	–	–
		50	24.0000	0.0000	1.1906	0.0000	6.2053	16.3988	–
	Grade 3	40	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.7568
		40	0.0000	0.2557	0.5924	0.0000	0.0000	0.0000	49.4339
		0	0.0000	0.2557	7.1731	24.0000	0.0000	0.0000	70.0000
	Grade 4	30	0.0000	0.0000	0.0000	0.0000	5.6070	0.0000	78.8094
		–	0.0000	0.0000	0.0000	0.0000	12.1877	15.8006	60.0000
	Lost	–	–	30.0000	0.0000	40.0000	39.7443	30.3835	0.0000
Purchase grade 2	Grade 2	100	–	–	–	–	–	–	–
		80	0.0000	–	–	–	–	–	–
		90	26.0000	0.0000	–	–	–	–	–
	Grade 3	40	0.0000	0.0000	0.0000	–	–	–	–
		30	0.0000	0.2557	2.5924	0.0000	–	–	–
		60	0.0000	0.2557	7.1731	26.0000	0.0000	–	–
	Grade 4	20	0.0000	0.0000	0.0000	0.0000	7.6070	0.0000	–
		–	0.0000	0.0000	0.0000	0.0000	12.1877	17.8006	100.000
	Lost	–	–	20.0000	60.0000	30.0000	39.7443	70.3835	0.0000
Purchase grade 3	Grade 3	90	–	–	–	–	–	–	–
		90	0.0000	–	–	–	–	–	–
		20	0.0000	0.2557	–	–	–	–	–
	Grade 4	10	0.0000	0.0000	0.0000	–	–	–	–
		–	0.0000	0.0000	0.0000	0.0000	–	–	–
	Lost	–	–	10.0000	20.0000	90.0000	89.7443	–	–
Purchase grade 4	Grade 4	50	–	–	–	–	–	–	–
		–	0.0000	–	–	–	–	–	–
	Lost	–	–	50.0000	–	–	–	–	–
Total Pulped →		–	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000	–

Table 8: Optimal pulping schedule for Example 8.2 (a four grade structured inventory with lifetime vector $\underline{T} = [4,3,3,2,\infty]^T$, with $d^p = 50$ and $r^p = 6$). The initial fruit stock for the pulping cycle is given in Table 5, while the fruit pricing structure is given in Table 7.