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STOCHASTIC ANALYSIS OF A TWO UNIT SYSTEM WITH VACATION FOR THE REPAIR FACILITY AFTER m REPAIRS

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ABSTRACT
Stochastic analysis of a two unit system with vacation period for the repair facility after the completion of m repairs is studied. All the underlying distributions are assumed to be non-Markovian. The reliability and availability analysis for such a system is studied. A numerical illustration is given.

Keywords: Repair facility, vacation period, reliability, avalability

1. INTRODUCTION

Two unit standby redundant repairable systems have attracted the attention of many applied probabilists and system analysts. In the literature available so far on such systems, it is clear that the repair facility is continuously available to attend to the repair of the failed units. However, it is reasonable to expect that a vacation might be needed for the repair facility after m repairs, before the next repair could be taken up. Such a vacation period certainly arises in many mechanical and electrical systems. This principle of a vacation period was first introduced by Subramanian and Sarma [2].

Recent developments in the modeling and analysis of highly reliable systems require the use of some more sophisticated models. According to Platis et al. [1] Electricité De France (EDF) used this for the homogeneous Markov approach to model an electrical substation in order to evaluate some measures of system performance. However, in this model, the underlying distributions are all non-Markovian, in the sense that, all the underlying distributions in this model are arbitrarily distributed.

In this paper, the concept of a vacation period for the repair facility after the completion of m repairs, and the arbitrary distributions described above have been introduced. The reliability and availability measures have been obtained using the regeneration point technique and product densities. A numerical example illustrates the results in section 7.

2. SYSTEM DESCRIPTION

(a) The system consists of two identical units: one unit is operating on-line and the other is kept in cold standby. Either unit performs the system functions satisfactorily.

(b) Switch is perfect and switchover is instantaneous.

(c) After the completion of m repairs, the repair facility is not available for a random time, denoted by 'vacation time' , the duration of which is governed by an arbitrarily distributed r.v. with p.d.f. $v(.)$.

(d) The life time of a unit while operating on-line is an arbitrarily distributed random variable (r.v.) with probability density function (p.d.f.) $f(.)$.

(e) The repair time of a unit is arbitrarily distributed with p.d.f. $g(.)$.

3. NOTATION

E_k	:	Event that the k th repair commences after completion of the vacation time; $k = 1, 2, ..., m$
D	:	Event that the system enters the down state
$f^{(n)}(t)$	:	$n-$ fold convolution of $f(t)$
$f(.), F(.), \bar{F}(.)$	:	p.d.f., c.d.f. and survivor function (s.f) of the lifetime of the unit
$g(.), G(.), \bar{G}(.)$	:	p.d.f., c.d.f. and s.f. of a repair time
$v(.), V(.), \bar{V}(.)$	:	p.d.f., c.d.f. and s.f. of the vacation time
$f^*(s)$	:	Laplace transform of $f(t)$

4. AUXILIARY FUNCTIONS

Let us define the following auxiliary functions, which will be used in the reliability and availability analysis.

We know that the events E_i, $i \geq 2$ are regenerative. Since E_1 can occur in two ways, it may or may not be a regenerative event.

Functions $f_j^r(t)$ and $f_j^a(t)$

For $j = 2, 3, ..., m-1$, let

$$f_j^r(t) = \lim_{\Delta \to 0} \frac{P\left[E_{j+1} \text{ in } (t, t+\Delta), N(D,t) = 0 \mid E_j \text{ at } t = 0\right]}{\Delta}$$

and

$$f_j^a(t) = \lim_{\Delta \to 0} \frac{P\left[E_{j+1} \text{ in } (t, t+\Delta) \mid E_j \text{ at } t = 0\right]}{\Delta}$$

At the epoch of occurrence of an E_j event, repair commences for a unit, while the other unit is just switched on-line.. Using probabilistic arguments we can find that

$$f_j^r(t) = f(t)G(t)$$

and

$$f_j^a(t) = f(t)G(t) + g(t)F(t)$$

Functions $f_{m_2}^r(t)$ and $f_{m_2}^a(t)$

Let

$$f_{m_2}^r(t) = \lim_{\Delta \to 0} \frac{P\left[E_2 \text{ in } (t, t+\Delta), N(D,t) = 0 \mid E_m \text{ at } t = 0\right]}{\Delta}$$

and

$$f_{m_2}^a(t) = \lim_{\Delta \to 0} \frac{P\left[E_2 \text{ in } (t, t+\Delta) \mid E_m \text{ at } t = 0\right]}{\Delta}$$

Figures 1 and 2 (Flowdiagram and Classification of events respectively) explain the various possibilities in order to derive
the expressions for $f_{m_2}^r(t)$ and $f_{m_2}^a(t)$, namely

$$f_{m_2}^r(t) = f(t)\left[g(t) \copyright V(t)\right] \copyright f(t)G(t) +$$

$$\int_{u_1=0}^{t} \int_{u_2=u_1}^{t} \int_{u_3=u_2}^{t} g(u_1)f(u_2)v(u_3-u_1)G(t-u_3)f(t-u_2)du_1du_2du_3$$

Considering all the 6 cases $\mathbb{A}$ to $\mathbb{F}$ in figure 2, we get

$$
\begin{aligned}
f_{m_2}^{a}(t) &= f(t)\left[g(t) © V(t)\right] © f(t)G(t) + f(t)\left[g(t) © V(t)\right] © g(t)F(t) + \\
&\int_0^t g(u_1)du_1 \int_{u_1}^t f(u_2)f(t-u_2)du_2 \int_{u_2}^t v(u_3-u_1)G\left(t-u_3\right)du_3 + \\
&\int_0^t g(u_1)du_1 \int_{u_1}^t f(u_2)F(t-u_2)du_2 \int_{u_2}^t v(u_3-u_1)g\left(t-u_3\right)du_3 + \\
&g(t)F(t) © f(t)\left[v(t) © G(t)\right] + g(t)F(t) © F(t)\left[v(t) © g(t)\right]
\end{aligned}
$$

Functions $f_{22}^{r}(t)$ and $f_{22}^{a}(t)$

Let

$$
f_{22}^{r}(t) = \lim_{\Delta\to 0} \frac{P\left[E_2\ in(t,t+\Delta), N(E_2,t)=0, N(D,t)=0 \mid E_2 \text{ at } t=0\right]}{\Delta}
$$

and

$$
f_{22}^{a}(t) = \lim_{\Delta\to 0} \frac{P\left[E_2 \text{ in } (t,t+\Delta), N(E_2,t)=0 \mid E_2 \text{ at } t=0\right]}{\Delta}
$$

$f_{22}^{r}(t)$ is the p.d.f. of the interval between two successive D avoiding E_2 events, and
$f_{22}^{a}(t)$ is the p.d.f. of the interval between two successive E_2 events.

Using probabilistic arguments, and observing that the events

$E_3, E_4, ..., E_m$ successively occur following an E_2 event, we have

$$
f_{22}^{r}(t) = \underset{j=2}{\overset{m-1}{\textcircled{\Pi}}} f_j^{r}(t) © f_{m_2}^{r}(t)
$$

$$
\underset{j=2}{\overset{k}{\textcircled{\Pi}}} f_j(t) = f_2(t) © f_3(t) © ... © f_k(t)
$$

5. RELIABILITY ANALYSIS

We observe that the E_2 events constitute a renewal process. For the sake of simplicity we assume that an E_2 event has occurred at $t = 0$.
Let

$$R(t) = P\left[N\left(D, t\right) = 0 \mid E_2 \text{ at } t = 0\right]$$

To derive an expression for the reliability $R(t)$ of the system, we consider the following mutually exclusive and exhaustive cases:

- no E_2 event occurs up to t, or
- at least one E_2 event occurs in $(0, t]$

Accordingly we have

$$\begin{aligned} R(t) = & \sum_{i=2}^{m-2} \underset{j=2}{\overset{i}{ⓟ}} f_j^r(t) \copyright \bar{F}(t) + \\ & \underset{j=2}{\overset{m-1}{ⓟ}} f_j^r(t) \copyright \left[\bar{F}(t) + f(t)G(t) \copyright \bar{F}(t)\right] + \\ & \sum_{n=1}^{\infty}\left[\left(f_{22}^r(t)\right)^{(n)}\right] \copyright \left[\sum_{i=1}^{m-2} \underset{j=2}{\overset{i}{ⓟ}} f_j^r(t) \copyright \bar{F}(t) + \right. \\ & \left. \underset{j=2}{\overset{m-1}{ⓟ}} f_j^r(t) \copyright \left\{\bar{F}(t) + f(t)G(t) \copyright \bar{F}(t)\right\}\right] \end{aligned}$$

Mean time to system failure (MTSF) can be obtained using the relation

$$\text{MTSF} = \int_0^\infty R(u)du = R^*(0)$$

6. AVAILABILITY ANALYSIS

This is defined as the 'probability that the system is able to operate within the tolerances at a given instant of time'. In symbols the pointwise availability is:

$$A(t) = P\,[\text{ system is up at } t \mid E_2 \text{ at } t = 0]$$

Using renewal theoretic arguments we get

$$\begin{aligned} A(t) \;=\; & \sum_{i=2}^{m-2} \underset{j=2}{\overset{i}{\textcircled{\Pi}}} f_j^a(t) \copyright \bar{F}(t) \; + \\ & \underset{j=2}{\overset{m-1}{\textcircled{\Pi}}} f_j^a(t) \copyright \left[\bar{F}(t) + f(t)G(t) \copyright \bar{F}(t) + F(t)g(t) \copyright \bar{F}(t)\right] + \\ & \sum_{n=1}^{\infty} \left[(f_{22}^a(t))^{(n)}\right] \copyright \left[\sum_{i=2}^{m-2} \underset{j=2}{\overset{i}{\textcircled{\Pi}}} f_j^a(t) \copyright \bar{F}(t) + \right. \\ & \left. \underset{j=2}{\overset{m-1}{\textcircled{\Pi}}} f_j^a(t) \copyright \left(\bar{F}(t) + f(t)G(t) \copyright \bar{F}(t) + g(t)F(t) \copyright \bar{F}(t)\right)\right] \end{aligned}$$

The steady state availability A_∞ can be obtained using the relation

$$A_\infty = \lim_{t\to\infty} A(t) = \lim_{s\to 0} sA^*(s)$$

Figure 1: Flowdiagram

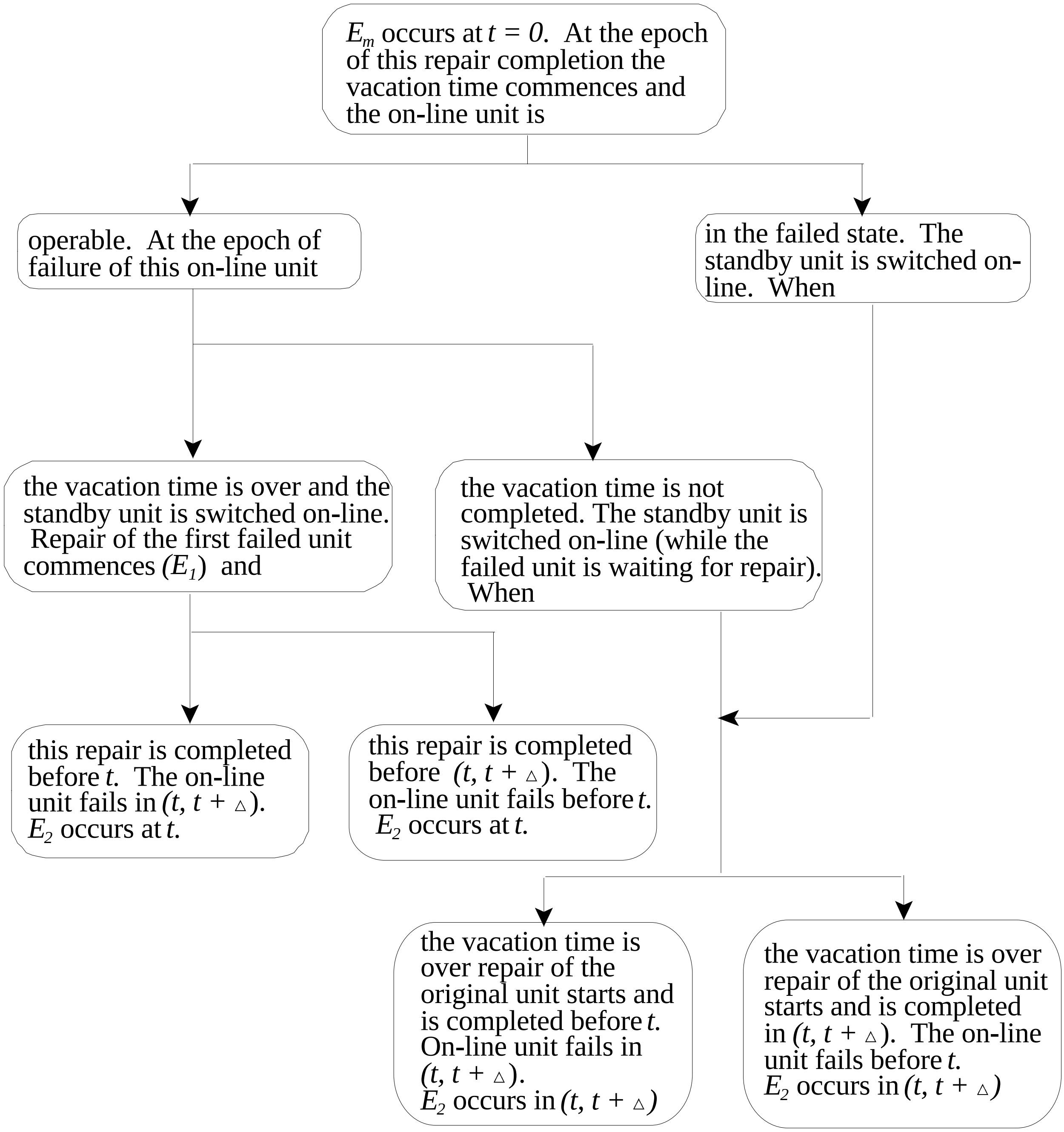
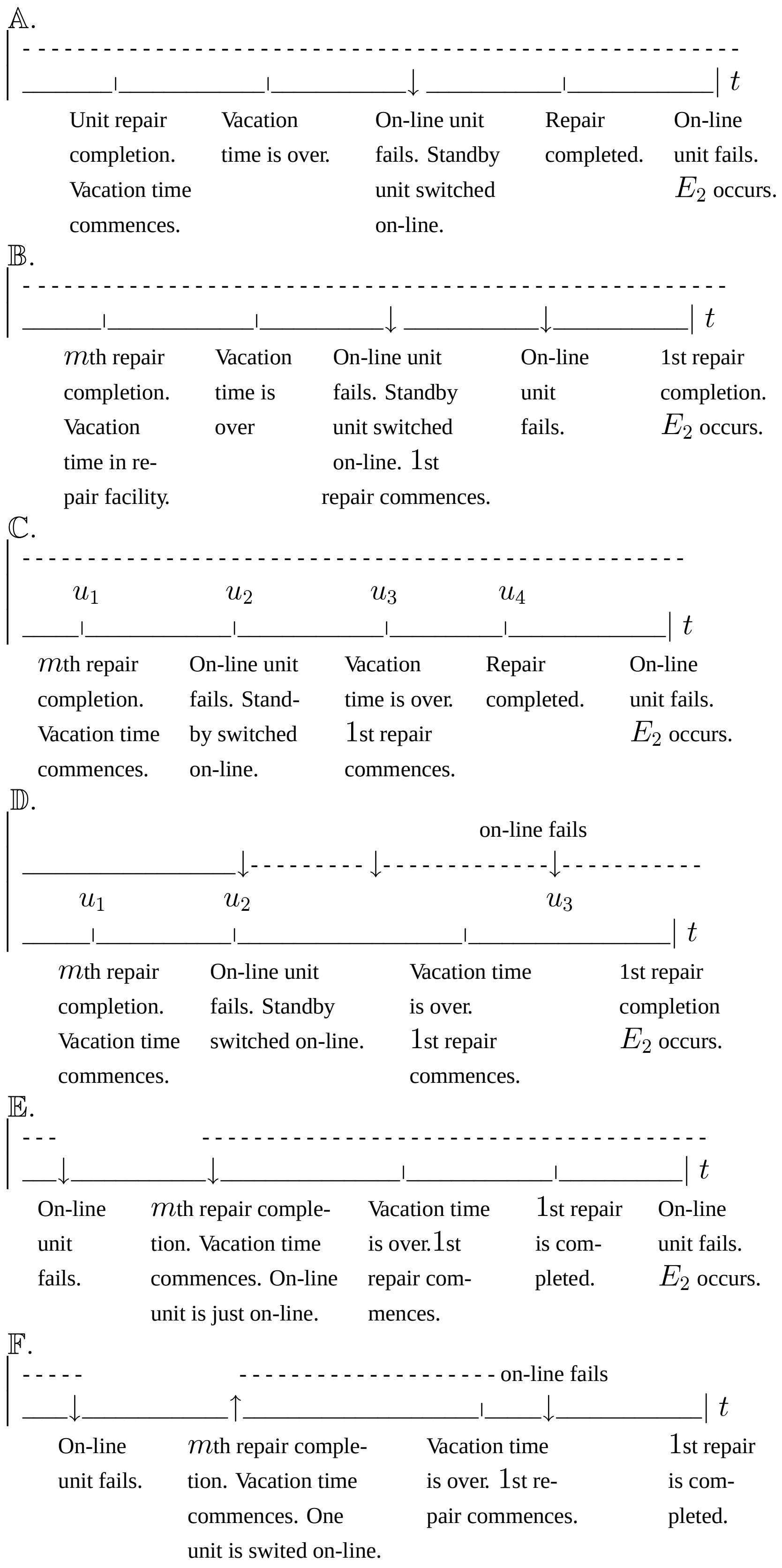


Figure 2: Classification of events



7. NUMERICAL ILLUSTRATION

A numerical example of some results obtained for the model are given in this section. For this purpose we assume that

$$f(t) = \lambda e^{-\lambda t}, \qquad \lambda > 0$$

$$g(t) = \mu e^{-\mu t}, \qquad \mu > 0$$

$$v(t) = d^2 t e^{-dt}, \qquad d > 0$$

The special case of $m = 3$ is considered.
Table 1 presents the mean time to system failure (MTSF) for different parametric values when $\lambda = \frac{1}{20}$

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TABLE 1

Rate of		
Vacation	Repair	MTSF
$d = \frac{1}{5}$	0.05	16.892
	0.08	25.129
	0.11	32.734
	0.14	39.897
	0.17	46.705
	0, 20	53.210
$d = \frac{1}{10}$	0.05	16.531
	0.08	24.030
	0.11	30.559
	0.14	36.375
	0.17	41.622
	0.20	46.396
$d = \frac{1}{15}$	0.05	16.230
	0.08	23.154
	0.11	28.898
	0.14	33.794
	0.17	38.040
	0.20	41.768
$d = \frac{1}{20}$	0.05	16.000
	0.08	22.504
	0.11	27.704
	0.14	31.996
	0.17	35.613
	0.20	38.710
$d = \frac{1}{25}$	0.05	15.825
	0.08	22.022
	0.11	26.840
	0.14	30.721
	0.17	33.924
	0.20	36.618