A stochastic model for an urea decomposition system

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Abstract

Availability is an important measure in describing the performance of a system. The availability of a decomposition process in an urea production system in the fertilizer industry is considered in this paper. The system contains four subsystems and is supported by a standby unit. An estimation study of the steady state availability of the system is performed and illustrated by means of a numerical example.

1 Introduction

The role and importance of (being able to measure) reliability has been a core issue in the Engineering industry over the last three decades. Reliability is of importance to both manufacturers and consumers. From the point of view of consumers and manufacturers, reliability provides quality and *vice versa*. The reliability measure is also very important, as improvement in reliability of a system is often achieved through quality analyses. While this measure is of great importance in industry, there are many situations where continuous failure free performance of a system, though desirable, may not be absolutely necessary. For instance, in a production industry possible interruptions may occur and can be tolerated. In such situations it may be eminently reasonable to introduce another measure called 'availability,' which denotes the probability that the system is functioning at any time instant.

In the process industry, like the fertilizer industry, one encounters many processes like synthesis decomposition, crystallization, prilling and recovery. See, for example, Kumar *et al.* (1991) and the UN Fertelizer Manual (1967).

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In an urea decomposition system a gas liquid mixture (urea, NH_3, CO_2, Biuret) flows from a reactor at 126°C into the upper part of a high-pressure decomposer where the flushed gasses are separated. The liquid falls through a sieve plate, which comes in contact with high temperature gas available from a boiler and a falling film heater. The process is repeated in a low-pressure absorber. The solution is further heated to 165°C in a falling film heater, which reduces the Biuret formation and hydrolysis of urea (see Figure 1 for the layout of a typical urea decomposition plant).

The overhead gasses from the high-pressure decomposition then go to the high-pressure absorber cooler. The liquid flows to the top of a low-pressure absorber and is cooled in a heat exchanger. Additional flushing of the solution takes place in the upper part of the low-pressure absorber so as to reduce the solution pressure from 17.5 to 2.5 kg/cm^2. The low-pressure absorber has four sieve trays and a packed bed. In the packed bed, the remaining ammonia is stripped off by means of CO_2 gas.

The overhead gases then go to the low-pressure absorber cooler, in which the pressure is controlled at 2.2 kg/cm^2. Most of the excess ammonia and carbonate is separated from the solution flowing to the gas separator. The gas separator has two parts:

1. the upper part functions at 105°C and 0.3 kg/cm^2 and here the remaining small amounts of ammonia and CO_2 are recovered by reducing the pressure; the heat of the solution is enough to vaporize these gasses.

2. The lower part comprises a packed section at 110°C and atmospheric pressure.

Air containing a small amount of ammonia and CO_2 is fed off from the gas absorber by an on/off gas blower so as to remove the remaining small amounts of ammonia and CO_2 present in the solution. Off gasses from the lower and upper parts are mixed and led to the off-gas condenser. The urea solution concentrated to 70–75% is finally fed to a crystallizer.

2 System description and notation

The complex system (urea plant in Figure 1) described in the previous section consists of four subsystems connected in series. The following assumptions are required to describe the system:

1. Subsystem (A_i) comprises two units. Unit A_1 is the boiler for the high-pressure absorber and A_2 is the falling filter heater for the low-pressure absorber. This subsystem (A_i) fails by failure of A_1 or A_2.

2. Subsystem B_i comprises two units in series. Unit B_1 is called the high-pressure absorber and unit B_2 is called the low-pressure absorber. Failure of either causes complete failure of the system.

3. Subsystem D, the gas separator, comprises only one unit, arranged in series with B_1 and B_2. Failure of unit D causes complete failure of the system.
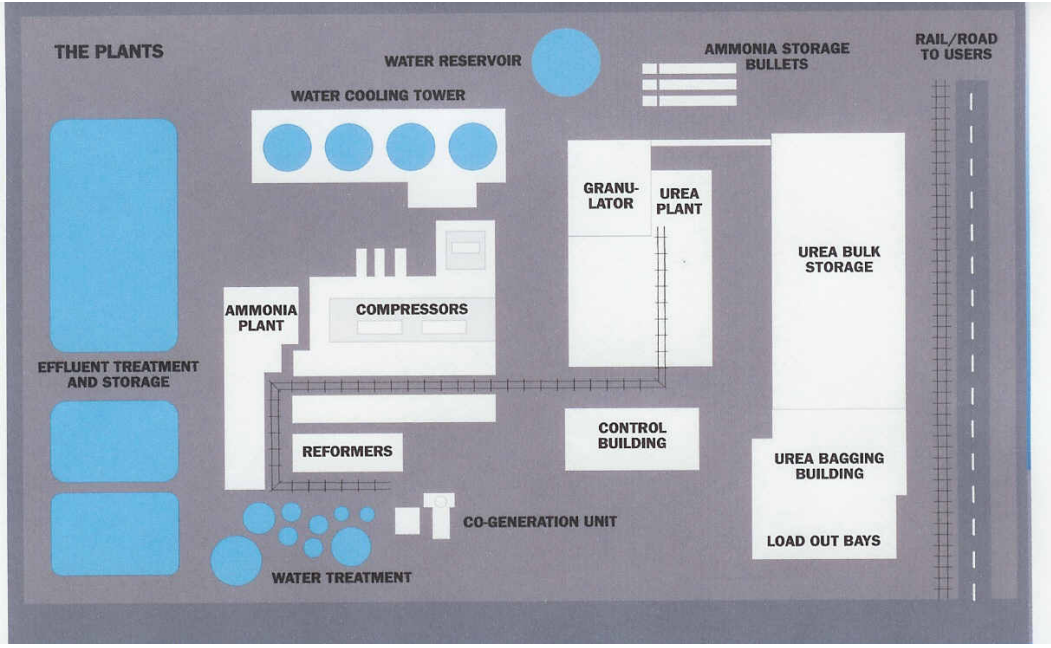
Figure 1: *Plan of an urea plant (by courtesy of Balance Kapuni, South Taranaki, New Zealand),*

4. Subsystem E_i the heat exchanger comprises one unit and another in standby mode. Failure occurs only when both units fail.

5. The life time of the units $(A_i, B_i, D, E_i; \quad i = 1, 2)$ are exponentially distributed random variables with parameters $\lambda_i; \quad i = 1, 2, \ldots, 6$.

6. The repair time of the units are exponentially distributed random variables with parameters $\mu_j; \quad j = 1, 2, \ldots, 6$.

7. Each unit is as good as new after repair.

8. Spare parts and the repair facility are always available.

9. The standby unit in E is of the same nature and capacity as the operating active unit.

10. The repair is performed at regular time intervals or at the time of failure. The repair includes replacement as well.

11. There is no common failure among subsystems.

12. State 0 indicates the operating state without using the standby unit and state 6 indicates the operating state using the standby state in subsystem E.

13. E_1 is the state of the system running at full capacity with a standby unit in subsystem E.
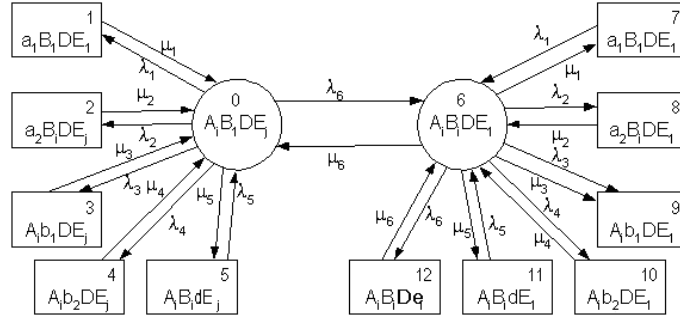
Figure 2: *The state transition diagram.*

3 Availability analysis of the system

Let $P_i(t)$ denote the probability that the system is in state i at time t and let $P_i = \lim_{t\to\infty} P_i(t)$.

Using Figure 2, the flow balance technique [2] is applied to find the steady state probabilities (P_i):

$$\left(\sum_{i=1}^{6} \lambda_i\right) P_0 = \sum_{i=1}^{6} \mu_i P_i, \tag{1}$$

$$\left(\sum_{i=1}^{6} \lambda_i + \mu_6\right) P_6 = \sum_{i=1}^{6} \mu_i P_{i+6} + \lambda_6 P_0, \tag{2}$$

$$\sum_{i=1}^{5} \mu_i P_i = \sum_{i=1}^{5} \lambda_i P_0; \quad i = 1, 2, \ldots, 6 \tag{3}$$

$$\sum_{i=1}^{6} \mu_i P_{i+6} = \sum_{i=1}^{6} P_6; \quad i = 1, 2, \ldots, 6 \tag{4}$$

$$\text{and} \quad \sum_{i=1}^{6} P_i = 1. \tag{5}$$

Solving the system of equations (1)–(5), we obtain P_i. The system measure A_∞, namely the probability that the system is up over the long run, may be obtained as

$$A_\infty = P_0 + P_6 = \frac{1 + \frac{\lambda_6}{\mu_6}}{1 + \left(1 + \frac{\lambda_6}{\mu_6}\right) \sum_{i=1}^{6} \frac{\lambda_i}{\mu_i}}.$$

4 Interval estimation for A_∞

Let $X_{i1}, X_{i2}, \ldots, X_{in}$ ($i = 1, 2, \ldots, 6$) be random samples of size n, each drawn from different exponential populations with failure rates λ_i. Similarly, let $y_{i1}, y_{i2}, \ldots, y_{in}$ ($i = 1, 2, \ldots, 6$) be random samples, each drawn from exponential populations with parameters

$\alpha_1=\alpha_2$	0	0	0	0.001	0.001	0.001	0.005	0.005	0.005
$\alpha_3=\alpha_4=\alpha_5$	0	0.005	0.01	0	0.005	0.01	0	0.005	0.01
$\alpha_6=0$	1	0.90909	0.83333	0.99602	0.90578	0.83056	0.98037	0.89284	0.81967
$\alpha_6=0.001$	0.99998	0.90907	0.83332	0.99600	0.90578	0.83055	0.98037	0.89284	0.81966
$\alpha_6=0.002$	0.99994	0.90903	0.83328	0.99595	0.90574	0.83052	0.98033	0.89280	0.81962
$\alpha_6=0.003$	0.99985	0.90897	0.83323	0.99587	0.90568	0.83046	0.98025	0.89274	0.81957
$\alpha_6=0.004$	0.99974	0.90888	0.83316	0.99577	0.90559	0.83039	0.98015	0.89266	0.81950
$\alpha_6=0.005$	0.99961	0.90877	0.83306	0.99563	0.90548	0.83029	0.98002	0.89254	0.81941
$\alpha_6=0.006$	0.99944	0.90863	0.83294	0.99546	0.90534	0.83018	0.97985	0.89241	0.81929
$\alpha_6=0.007$	0.99924	0.90846	0.83280	0.99526	0.90517	0.83004	0.97966	0.89255	0.81916
$\alpha_6=0.008$	0.99901	0.90827	0.83264	0.99503	0.90498	0.82988	0.97944	0.89207	0.81901
$\alpha_6=0.009$	0.99875	0.90806	0.83247	0.99478	0.90477	0.82970	0.97919	0.89186	0.81883

Table 1: *Availability measure,* A_∞ *(*$\beta_1 = \beta_2 = 0.5$*;* $\beta_3 = \beta_4 = 0.2$*;* $\beta_5 = 0.1$*;* $\beta_6 = 0.25$*).*

$\alpha_1=\alpha_2$	0	0	0	0.001	0.001	0.001	0.005	0.005	0.005
$\alpha_3=\alpha_4=\alpha_5$	0	0.005	0.01	0	0.005	0.01	0	0.005	0.01
$\alpha_6=0$	1	0.90900	0.83333	0.99668	0.90634	0.83102	0.98361	0.89552	0.82192
$\alpha_6=0.001$	0.99998	0.90908	0.83332	0.99666	0.90633	0.83101	0.98359	0.89551	0.82191
$\alpha_6=0.002$	0.99994	0.90904	0.83329	0.99661	0.90629	0.83098	0.98355	0.89547	0.82187
$\alpha_6=0.003$	0.99986	0.90894	0.83323	0.99654	0.90623	0.83093	0.98347	0.89541	0.82182
$\alpha_6=0.004$	0.99975	0.90888	0.83316	0.99643	0.90614	0.83085	0.98336	0.89532	0.82175
$\alpha_6=0.005$	0.99961	0.90877	0.83316	0.99629	0.90602	0.83075	0.98323	0.89521	0.82165
$\alpha_6=0.006$	0.99944	0.90863	0.83306	0.99612	0.90588	0.83064	0.98306	0.89507	0.82154
$\alpha_6=0.007$	0.99924	0.90846	0.83294	0.99592	0.90572	0.83050	0.98287	0.89491	0.82140
$\alpha_6=0.008$	0.99901	0.90827	0.83280	0.99569	0.90553	0.83034	0.98265	0.89473	0.82125
$\alpha_6=0.009$	0.99875	0.90806	0.83247	0.99544	0.90532	0.83016	0.98240	0.89452	0.82107

Table 2: *Availability measure,* A_∞ *(*$\beta_1 = \beta_2 = 0.6$*;* $\beta_3 = \beta_4 = 0.2$*;* $\beta_5 = 0.1$*;* $\beta_6 = 0.25$*).*

| $\alpha_1=\alpha_2$ | 0 | 0 | 0 | 0.001 | 0.001 | 0.001 | 0.005 | 0.005 | 0.005 |
$\alpha_3=\alpha_4=\alpha_5$	0	0.005	0.01	0	0.005	0.01	0	0.005	0.01
$\alpha_6=0$	1	0.92308	0.85714	0.99715	0.92065	0.85505	0.98592	0.91106	0.84677
$\alpha_6=0.001$	0.99998	0.92306	0.85713	0.99714	0.92064	0.85504	0.9859	0.91105	0.84676
$\alpha_6=0.002$	0.99994	0.92302	0.85710	0.99709	0.9206	0.85500	0.98585	0.91101	0.84673
$\alpha_6=0.003$	0.99986	0.92296	0.85704	0.99701	0.92053	0.85494	0.98578	0.91094	0.84667
$\alpha_6=0.004$	0.99975	0.92286	0.85696	0.9969	0.92044	0.85486	0.98567	0.91085	0.84659
$\alpha_6=0.005$	0.99961	0.92274	0.85685	0.99676	0.92032	0.85476	0.98553	0.91074	0.84649
$\alpha_6=0.006$	0.99944	0.9226	0.92260	0.99659	0.92017	0.84637	0.98537	0.91060	0.84637
$\alpha_6=0.007$	0.99924	0.92243	0.92243	0.99639	0.92000	0.84623	0.98517	0.91043	0.84623
$\alpha_6=0.008$	0.99901	0.92223	0.92223	0.99617	0.91981	0.84606	0.98495	0.91024	0.84606
$\alpha_6=0.009$	0.99875	0.92201	0.92201	0.99591	0.91969	0.84588	0.98470	0.91003	0.84588

Table 3: *Availability measure,* A_∞ *(*$\beta_1 = \beta_2 = 0.7$*;* $\beta_3 = \beta_4 = 0.3$*;* $\beta_5 = 0.1$*;* $\beta_6 = 0.25$*).*

$\alpha_1=\alpha_2$	0	0	0	0.001	0.001	0.001	0.005	0.005	0.005
$\alpha_3=\alpha_4=\alpha_5$	0	0.005	0.01	0	0.005	0.01	0	0.005	0.01
$\alpha_6=0$	0.99998	0.93023	0.86957	0.99751	0.92807	0.86768	0.98765	0.91954	0.86022
$\alpha_6=0.001$	0.99994	0.93022	0.86955	0.99749	0.92806	0.86767	0.98764	0.91953	0.8602
$\alpha_6=0.002$	0.99986	0.93018	0.86952	0.99744	0.92802	0.86763	0.98759	0.91949	0.86017
$\alpha_6=0.003$	0.99975	0.93011	0.86946	0.99736	0.92795	0.86757	0.98752	0.91942	0.86011
$\alpha_6=0.004$	0.99975	0.93001	86937	0.99726	0.92786	0.86749	0.98741	0.91933	0.86003
$\alpha_6=0.005$	0.99961	0.92989	0.8627	0.99712	0.92774	0.86738	0.98727	0.91921	0.85992
$\alpha_6=0.006$	0.99944	0.92975	0.86914	0.99695	0.92759	0.86726	0.98711	0.91906	0.858
$\alpha_6=0.007$	0.99924	0.92957	0.86899	0.99675	0.92742	0.86711	0.98691	0.9189	0.8597
$\alpha_6=0.008$	0.99901	0.92937	0.86882	0.99652	0.92722	0.86893	0.98669	0.9187	0.85948
$\alpha_6=0.009$	0.99875	0.92915	0.86862	0.99626	0.927	0.86674	0.98644	0.91848	0.85929

Table 4: *Availability measure, A_∞ ($\beta_1 = \beta_2 = 0.8$; $\beta_3 = \beta_4 = 0.4$; $\beta_5 = 0.1$; $\beta_6 = 0.25$).*

$\alpha_1=\alpha_2$	0	0	0	0.001	0.001	0.001	0.005	0.005	0.005
$\alpha_3=\alpha_4=\alpha_5$	0	0.005	0.01	0	0.005	0.01	0	0.005	0.01
$\alpha_6=0$	1	0.93458	0.87719	0.99778	0.93264	0.87549	0.98901	0.92497	0.86873
$\alpha_6=0.001$	0.99998	0.93457	0.87718	0.99777	0.93263	0.87547	0.989	0.92496	0.86871
$\alpha_6=0.002$	0.99994	0.93452	0.87714	0.99772	0.93259	0.87544	0.98895	0.92492	0.86868
$\alpha_6=0.003$	0.99986	0.93446	0.87708	0.99764	0.93252	0.87538	0.98887	0.92485	0.86862
$\alpha_6=0.004$	0.99975	0.93436	0.871	0.99753	0.93242	0.87529	0.98876	0.92476	0.86854
$\alpha_6=0.005$	0.99961	0.93424	0.87689	0.99739	0.9323	0.87519	0.98863	0.92464	0.86843
$\alpha_6=0.006$	0.99944	0.93409	0.87676	0.99722	0.93215	0.87506	0.98846	0.92449	0.8683
$\alpha_6=0.007$	0.99924	0.93391	0.87661	0.99702	0.93198	0.8749	0.98827	0.92432	0.86815
$\alpha_6=0.008$	0.99901	0.93371	0.87643	0.9968	0.93178	0.87473	0.98804	0.92413	0.86798
$\alpha_6=0.009$	0.99875	0.93349	0.87623	0.99654	0.93156	0.87453	0.98779	0.92391	0.86778

Table 5: *Availability measure, A_∞ ($\beta_1 = \beta_2 = 0.9$; $\beta_3 = \beta_4 = 0.5$; $\beta_5 = 0.1$; $\beta_6 = 0.25$).*
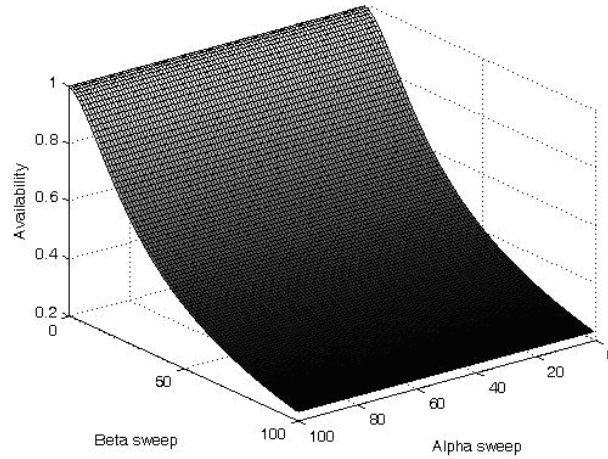
Figure 3: *Availability measure, A_∞, for different values of α (failure-free operation time) and β (repair time) values.*

μ_i. Since λ_i is a parameter of the exponential distribution, an estimate may be found for λ_i or for $1/\lambda_i = \alpha_i$ (say), which is equal to the mean value of failure-free time. Let

$$\alpha_i = \frac{1}{\lambda_i} \quad \text{and} \quad \beta_i = \frac{1}{\mu_i}.$$

Tables 1 to 5 and Figure 3 show values of the above availability function, A_∞ for different values of the parameters α_i and β_i. Then the maximum likelihood estimates (MLE) of α_i and β_i are given by

$$\frac{1}{n} \sum_{j=1}^{n} X_{ij} = \bar{x}_i \quad \text{and} \quad \frac{1}{n} \sum_{j=1}^{n} Y_{ij} = \bar{y}_i$$

respectively. Hence

$$\hat{A}_\infty = \frac{1 + \bar{y}_6/x_6}{1 + (1 + \bar{y}_6/\bar{x}_6) \sum_{i=1}^{6} \bar{y}_i/\bar{x}_i}.$$

Applying the multivariate central limit theorem (See, for example, Rao (1973)), yields $\sqrt{n}(\bar{x} - \theta) \xrightarrow{D} N_6(0, \Sigma)$ as $n \to \infty$, where $\bar{x} = (\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{x}_4, \bar{x}_5, \bar{x}_6, \bar{y}_1, \bar{y}_2, \bar{y}_3, \bar{y}_4, \bar{y}_5, \bar{y}_6)$ and $\theta = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6)$. The dispersion matrix $\Sigma = (\sigma_{ij})_{12 \times 12}$ is given by $\Sigma = \text{diag}(\alpha_1^2, \dots, \alpha_6^2, \beta_1^2, \dots, \beta_6^2)$. Hence, $\sqrt{n}(\hat{A}_\infty - A_\infty) \xrightarrow{D} N_6(0, \sigma^2(\theta))$, where

$$\sigma^2(\theta) = \sum_{i=1}^{6} \left(\frac{\delta A_\infty}{\delta \alpha_i} \right)^2 \delta_{ii} + \sum_{i=1}^{6} \left(\frac{\delta A_\infty}{\delta \beta_i} \right)^2 \delta_{ii}$$

and where $\hat{\theta} = (\bar{\alpha}_1, \bar{\alpha}_2, \bar{\alpha}_3, \bar{\alpha}_4, \bar{\alpha}_5, \bar{\alpha}_6, \bar{\beta}_1, \bar{\beta}_2, \bar{\beta}_3, \bar{\beta}_4, \bar{\beta}_5, \bar{\beta}_6)$, which entails that $\hat{\sigma}^2 = \sigma^2(\theta)$ is a consistent estimator of $\sigma^2(\theta)$. Therefore by Slutzky's theorem (see, for example, Slutzky (1928)),

$$\frac{\sqrt{n}(\hat{A}_\infty - A_\infty)}{\hat{\sigma}} \xrightarrow{D} N(0, 1) \text{ as } n \to \infty$$

or,

$$P \left[-\kappa_{\frac{\alpha}{2}} \leq \frac{\sqrt{n} \left(\hat{A}_\infty - A_\infty \right)}{\hat{\sigma}} \leq \kappa_{\frac{\alpha}{2}} \right] = 1 - \alpha.$$

5 Numerical illustration

The $\alpha = 95\%$ and $\alpha = 99\%$ confidence intervals for different simulated samples are presented in Table 6. Observe that the steady state availability decreases if n increases.

6 Conclusion

The availability of equipment used for the decomposition process in a urea production system was considered in this paper. The system consists of four subsystems, with a standby unit in one of the sub-systems. The long run availability of the system is calculated, and asymptotic confidence limits are obtained for the steady-state availability. The results clearly show that, as the repair time increases, the steady state availability decreases.

		$\alpha = 95\%$	$\alpha = 99\%$
$n = 100$	20	(0.79414; 0.96586)	(0.76702; 0.99298)
	40	(0.62674; 0.78366)	(0.60196; 0.80824)
	60	(0.54593; 0.68317)	(0.52533; 0.69537)
	80	(0.50775; 0.61515)	(0.48552; 0.62088)
	100	(0.47816; 0.56924)	(0.45556; 0.57544)
$n = 200$	20	(0.81928; 0.94072)	(0.80008; 0.95992)
	40	(0.64986; 0.76074)	(0.63234; 0.77826)
	60	(0.57289; 0.66421)	(0.55843; 0.67867)
	80	(0.52347; 0.59943)	(0.51147; 0.61143)
	100	(0.49148; 0.55592)	(0.48128; 0.56612)
$n = 2000$	20	(0.86080; 0.89920)	(0.85468; 0.90532)
	40	(0.68790; 0.72270)	(0.67998; 0.73062)
	60	(0.60409; 0.63301)	(0.59953; 0.63757)
	80	(0.54945; 0.57345)	(0.54567; 0.57723)
	100	(0.51356; 0.53384)	(0.51032; 0.53708)

Table 6: *Confidence limits for A_∞ (For $\alpha_1 = \alpha_2 = 0$; $\alpha_3 = \alpha_4 = \alpha_5 = 0$; $\alpha_6 = 0.001 \beta_1 = \beta_2 = 0.5$; $\beta_3 = \beta_4 = 0.2$; $\beta_5 = 0.1$; $\beta_6 = 0.25$).*

Acknowledgements

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