ORiON, Vol. 17, No. 1/2, pp. 29-54 *ISSN 0259-191-X*

ASSETS, LIABILITIES AND RISK

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ABSTRACT
Financial economists and actuaries do not always talk the same language. One particular difference of concern to actuaries is the method of treatment (or non-treatment) of the liabilities of an investor in the portfolio selection problem. Another difference relates to the way in which liabilities are valued. In this paper, these differences are discussed and possible ways forward are suggested.

1. INTRODUCTION

In the development of portfolio theory, Markowitz (1952, 1959) made no reference to the liabilities of the investor. The reasons were presumably twofold: in the first place, it could be assumed that a liability was merely a short position in one of the assets in the opportunity set, and secondly it could be assumed that that asset, like all the others in that set, was tradable. The Capital Asset Pricing Model (CAPM) (Tobin, 1958; Sharpe, 1964; Lintner, 1965; Mossin, 1966) was also developed without reference to the liabilities of investors, mainly because its authors were concerned with capital assets (i.e. assets in non-zero supply in the economy) or at least shares and other financial instruments providing entitlement to the proceeds of such assets. The use of the CAPM to define a 'market price of risk' resulted in a concept of risk without reference to the investor's liabilities. Subsequent measures of risk such as Sharpe's (1966) measure, Treynor's (1966) measure, Jensen's (1968, 1969) measure and the appraisal ratio (Bodie, Kane & Marcus, 1996: 780) follow suit.

Borch (1990) seeks to bridge the gap between financial economists and actuaries with regard to problems related to insurance. But his interest in the portfolio selection problem relates more to insurance and reinsurance portfolios than to capital asset portfolios. And his interest

in the pricing of liabilities relates more to the development of explanatory models of market prices than to the determination of market-based values for complex portfolios of liabilities that are not explicitly priced in the market.

This paper discusses the importance of the liabilities of an investor in the discussion of risk, as well as the quantification of risk for the purposes of the valuation of the liabilities. Section 2 considers the former matter from the perspective of financial economics and section 3 from the point of view of fund management. Section 4 discusses the use of benchmark portfolios based on asset and liability modelling. Section 5 compares this approach with that of the CAPM. In section 6, consideration is given to the determination of the value of the liabilities. In order to justify reference to the liabilities of the financial institution, as opposed to the preferences of prospective beneficiaries, in the asset allocation problem, the principal-agent problem is discussed in section 7, as well as the trusteeship function. Section 8 concludes.

2. THE PLACE OF LIABILITIES IN FINANCIAL ECONOMICS

Both portfolio theory and the CAPM are based on the expected utility theory of Von Neumann & Morgenstern (1947). Expected utility theory, in turn, is based on the distribution of the outcomes and the utility function of the decision-maker. Now suppose, for simplicity, that the decision-maker is an investor with assets and liabilities maturing at a specified time horizon. It must be assumed that the decision-maker will be indifferent between an extra rand of asset proceeds and one rand less of liability payments at the time horizon. If the future proceeds of assets available to the decision-maker are correlated with the future payments for which it will become liable, it is the net future proceeds whose distribution must be considered. All else being equal, if the investor invests in assets that are positively correlated with its liabilities, the risks of low net proceeds at the time horizon will be reduced and vice versa. Conversely, if the investor invests part of its wealth in assets that are positively correlated with its other assets, the risks of low proceeds at the time horizon are increased and vice versa. And indeed, these results follow from portfolio theory.

Suppose, for example, that an investor has fixed exposure k (which may be negative) to security 1, and αl and $(1 - \alpha)l$ to securities 2 and 3 respectively, where k and l (> 0) are specified constants and α is the decision variable. Suppose that the values of the securities at the time horizon are jointly distributed with mean

$$\mu = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{pmatrix}$$

and covariance matrix

$$\Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix}.$$

Suppose also that the investor has a quadratic utility function.

Then it may be shown that the value of α that maximises the investor's expected utility is

$$\alpha = a + b\mu_1 + ck(\sigma_{13} - \sigma_{12})$$

where *a*, *b* and *c* are functions of $\mu_2, \mu_3, \sigma_{22}, \sigma_{23}, \sigma_{33}$, and $c > 0$.

Thus, if *k* is positive (so that security 1 is an asset), then the optimal exposure to security 2 increases as the covariance of the values of securities 1 and 3 increases relative to that of securities 1 and 2, and vice versa. If *k* is negative (so that security 1 is a liability) then the converse holds.

The assumption of quadratic utility is used to simplify the above analysis. Utility functions of higher order would introduce higher moments than covariances into the relationships involved. But the above analysis is sufficient to show that, in general, it is the expected utility of the entire net wealth of the decision-maker that must be maximised, not the expected utility of that portion of the decision-maker's gross wealth which has been allocated to a particular investment manager.

Investment practitioners frequently abuse the theory by ignoring investors' liabilities. Typically, linked internal rates of return on investments are determined for investment performance measurement and compared either with the return on an appropriate index or with the returns achieved by other investment managers. (Adams, 1989: 310-6; Bodie, Kane & Marcus, 1996: 778) These comparisons are invariably made without reference to the investors' liabilities. Even where risk-adjusted investment returns are compared, liabilities are typically ignored. (Bodie, Kane & Marcus, 1996: 779-88) Academics and other researchers have followed suit (e.g., in South Africa, Patrick & Ward, 1996 and Swart, 1999).

Adams (1989: 9) distinguishes between financial economists' and actuaries' definitions of risk. Whereas a financial economist, he suggests, might define risk to be 'uncertainty of future returns' on investments, an actuary would define it with reference to the investor's 'ability to meet liabilities when they fall due'. This distinction may be true in practice, but if so it is not portfolio theory that is at fault. Thomson (1998) defended the use of expected utility theory for normative purposes in the context of defined-contribution retirement funds. Although mean-variance analysis has its faults—and the necessary caveats are clearly stated and discussed in Markowitz (1959)—it is still useful as a first approximation to the portfolio selection problem, provided the liabilities of the investor are taken into account. The necessary caveats were similarly stated by Sharpe (1964) with regard to the CAPM, but they have been largely ignored by practitioners, despite numerous findings that the underlying assumptions do not hold in practice. This matter is discussed further in section 5 below.

3. THE PLACE OF LIABILITIES IN FUND MANAGEMENT

In investment management parlance, 'fund management' refers only to the assets of a fund. In practice the liabilities of a fund may be beyond the control even of the trustees, let alone the investment managers. But this does not mean that the liabilities can be ignored in the management of the assets.

Leibowitz (1987) introduces the concept of a 'return on liabilities', which can be calculated retrospectively in much the same way as a return on assets. Whilst this concept has some explanatory value, it does not mean anything on its own. It can be compared with the return on assets, but without knowledge of the relative values of the assets and liabilities the net effect is unclear. What is needed, therefore, is to focus on the surplus (i.e. the value of the assets less the value of the liabilities) in monetary terms—in other words, on the net wealth of the fund.

If an investor's net wealth $W(t)$ is positive at time $t = 0$ then its value at time $t > 0$ is a positive linear function of the return $I(t)$ earned up to that time; i.e.

$$W(t) = W(0)\{1 + I(t)\}.$$

Under these circumstances it would therefore be consistent with portfolio theory and the CAPM to measure investment performance in terms of the returns earned on net wealth.

However, contrary to the assumptions of portfolio theory and the CAPM, liabilities cannot be regarded as short positions in tradable assets. Otherwise, investment managers could be given control over the liabilities of investors as well as their assets, and financial institutions would be unnecessary. In the first place, if a liability is transferred from the original counterparty to another, the counterparty risks associated with the liability are changed, and the value of the liability changes accordingly. For example, if a retirement fund transfers its liability for a final-salary pension to a third party, the third party may be unable to meet the liability when it becomes payable.

On the other hand, the moral hazard associated with the liability may increase enormously on such transfer, thus increasing its value. This is particularly true where the moral hazard associated with a particular counterparty is controlled by a convergence of interest between the institution and that counterparty. For example, if a retirement fund transfers its liability for a final-salary pension to a third party, the employer may substantially increase the employee's salary at the end of the employee's career, thus increasing the value of the liability without any cost to the employer. In this case the institution is the retirement fund and the counterparty is the employer. The increase in the value of the liability arises because the counterparty no longer has an interest in minimising the cost of funding the benefits.

In terms of the Pension Funds Act (Act no. 24 of 1956 as amended), a retirement fund member may not cede or assign his or her rights to benefits from a fund, and a transfer of liabilities from a retirement fund is subject to strict regulation.

Retirement funds may be able to insure some of their benefits—specifically those for which there is little or no moral hazard—but this leaves the core liabilities of a final-salary retirement fund, namely the normal retirement benefits, uninsurable and untradable.

The liabilities of life offices are less circumscribed. Life offices are able to reinsure their liabilities to a large extent. This means that, within the reinsurance market, they are tradable. However, the reinsurance market is generally distinct from the securities markets. Although reinsurers are increasingly providing products that modify the investment risks of insurers, different managers are involved in the management of the assets of the life office and must be separately mandated and measured.

In retirement funds, thanks to the popularity of split funding, not only does an investment manager not have control over the liabilities, it may not even have control over all the assets. How, then, do we take cognisance of the entire net wealth of the fund in managing its risks?

4. BENCHMARK PORTFOLIOS BASED ON ASSET AND LIABILITY MODELLING

In the light of the above considerations, it is therefore becoming increasingly widely advocated that investment managers should be measured against a benchmark portfolio constructed to reflect the liabilities of the investor. A revised regulation 28 has been drafted by the Financial Services Board to allow for this process in terms of the Pension Funds Act. And the Actuarial Society of South Africa is currently working on a new guidance note on this subject for actuaries advising the trustees of retirement funds. One of the research projects of the Actuarial Society relates to the capital adequacy requirements of South African life offices. This project will necessitate the stochastic modelling of the assets and liabilities so as to develop measures of capital adequacy.

Booth et al (1999: 98-103) trace the development of asset and liability modelling, which has now become a standard method of optimising a portfolio in the presence of liabilities. The optimisation problem was first set out by Wise (1984) and was further developed by Wilkie (1985) and Sherris (1992).

In broad outline, the actuarial contribution to portfolio optimisation followed the portfolio theory of Markowitz (op. cit.), but allowed explicitly for the investor's liabilities. Initially, in Wise and Wilkie (op. cit.), an efficient frontier was developed in mean-variance space defined as the mean and variance of the surplus at a suitable time horizon. Panjer (1998: 397-405) gives a modification of this approach, also in mean-variance space. Sherris (op. cit.) subsequently generalised the problem beyond the constraints of mean-variance analysis.

This also permitted the use of non-quadratic utility functions and non-elliptical distributions of returns on investments and growth of liabilities. Because the time horizons of investors other than banks, general insurers and unit trusts typically extend over many years, multivariate stochastic models of investment returns, and of variables (such as inflation rates) driving investors' liabilities, were developed. These include Wilkie's (1986, 1995) and Smith's (1996) model, the TY model (Yakoubov, Teeger & Duval, 1999) and the Whitten & Thomas

(2000) model, all for the United Kingdom. Models developed for other countries include Carter (1991) for Australia, Sharp (1994a, 1994b) and Deaves (1994) for Canada and Thomson (1996) for South Africa. There is room for improvement in all the published models to incorporate recent developments in modelling techniques. Numerous unpublished models have also been developed in-house by commercial organisations.

Because of the complexity of the liabilities of long-term investors and of the stochastic models, it is not generally possible to obtain solutions to the optimisation problem in closed form. However, the models facilitate the generation of pseudo-random samples of the surplus at an appropriate time horizon, which may be used to select portfolios (and other decision variables) to maximise the expected utility of that surplus. In practice, the utility functions of the investors (who may, for example, be a board of trustees[1]) is indeterminate. Under these circumstances it may be necessary to produce an efficient frontier of the surplus at the time horizon in mean-variance space (ignoring the effects of higher moments) and to present alternative portfolios to the trustees along that frontier.

Since it is the trustees who must decide on the investment allocations, it is the trustees who must define the objective function for the ALM, i.e. the surplus or funding ratio at a time horizon selected by them. The most natural definition of the surplus is therefore the surplus on the basis used for funding purposes, taking into account the principles discussed above. This is further discussed in section 6. The most natural definition of the time horizon is the date on which the surplus will next be determined.

Instead of the variance, risk may be presented as a generalised lower partial moment:

$$\frac{1}{J}\sum_{j=1}^{J}\{\max(K - S_j, L)\}^n$$

where: J is the number of simulations;

S_j is the surplus at the time horizon on the jth simulation; and

K, L and n may be suitably selected subject to the constraints:

$L \geq 0$; and

$n > 0$.

[1] In this paper, reference is made to the investment decision-makers as 'trustees'. This is to reflect the fact that, contrary to the assumptions of elementary economic models, investment decisions are generally the responsibility of persons or organisations to whom such responsibility has been explicitly or implicitly entrusted.

(K would normally represent the targeted surplus, L would represent a level below which risk is irrelevant, i.e. the value about which the lower partial moment is determined, and n would represent the order of the moment selected.)

Alternatively, it may be presented as a lower quantile of the surplus at the time horizon, analogously to value at risk (Beckström & Campbell, 1995).

The design of measures of risk is in fact a process currently receiving attention, but is considered to be outside the scope of this paper. Various properties have been proposed to constitute a 'coherent measure of risk'. (Artzner et al, 1999).

Whatever measure of risk is used, it should allow for the liabilities. An efficient frontier may then be presented to the trustees in risk-reward space. The trustees' initial selection may then be used to determine their risk aversion, thus defining an approximation to their utility function, which may be used to obtain an approximate solution. Empirical research in this field would be valuable.

The portfolio thus selected may be used as a benchmark for defining the investment manager's mandate, and for the measurement of the investment performance. Investment managers may then be compared with reference to the extent to which they outperform their benchmarks relative to the degree of flexibility allowed in their mandate to depart from those benchmarks.

In effect, the benchmark becomes a notional liability for the investment manager. Although the investment manager cannot trade it, it comprises tradable assets, and therefore provides a basis for investment risk management that is both fair to the investment manager and consistent with the liabilities of the financial institution.

5. ALM VERSUS CAPM

As outlined above, ALM may be used as a tool for portfolio selection in the same way as in Markowitz (*op. cit.*). And in the same way, ALM does not attempt to address the question how capital assets would be priced if all investors used the same model.

The CAPM extends Markowitz portfolio theory by considering what happens if all investors apply that theory, with the same assumptions about the means and variances of returns. It derives a formula for the market price of risk, and hence for the market price of any capital asset, in terms of the distribution of its return.

It may be noted that the asset and liability models developed for actuarial use are based on the concepts of expected utility theory, or at most on those of portfolio theory. The concepts of the CAPM are not invoked. The reasons for this are threefold.

In the first place, it has been argued by Wilkie (unpublished) that, whereas investment returns may show no evidence of autoregression or moving-average effects over short time intervals such as the daily, weekly or monthly intervals typically used for the CAPM, they may well show such effects over the longer intervals such as quarterly or annual intervals typically used for the stochastic modelling of long-term financial institutions. Fama & French (1988) also record findings by financial economists that returns on shares are predictable, particularly for longer time horizons. Thus the efficient markets hypothesis (Fama, 1965) on which the CAPM depends must be rejected for long-term financial institutions. Individual investors can take views that differ from the market, and it is sub-optimal not to do so.

Secondly, it has been shown by Wilkie (1997) that the CAPM fails in a multi-currency world. First he shows that, for investors in two countries, each holding domestic cash and domestic and foreign (i.e. the other country's) shares, the risk-free assets and the market portfolios are different. He then shows that, if the markets in the two countries are of different sizes, equilibrium can be obtained only if the prices of shares change.

Thirdly, the advantage of the CAPM lies in its simplicity. Although the assumptions of the CAPM have been relaxed to accommodate reality more credibly, each relaxation results in additional complexity, thus detracting from the simplicity of the model. In fact, Ross (1977) shows that, when both riskless lending and borrowing and short sales are disallowed, one cannot derive a simple general equilibrium expression for the expected return on a risky asset.

In defence of the CAPM, it must be recognised that, as pointed out by Sharpe (1964), "the proper test of a theory is not the realism of its assumptions but the acceptability of its implications." Much work has been done on the testing of those implications. (Elton &

Gruber, 1995: 341-62) The results are inconclusive. In fact Roll (1977) finds that the CAPM cannot be tested unless the entire global market portfolio (including unmarketable assets such as human capital) can be identified. Nevertheless, although the assumptions are not necessarily realised by the market, it is still of value in determining approximations both to the market value of an investor's liabilities and to the efficient frontier in mean-standard-deviation space. Particularly valuable in this regard are the extensions of the CAPM to allow for no riskless borrowing, non-marketable assets (and liabilities), heterogeneous expectations, multiple periods, inflation risk and multiple betas. (Elton & Gruber: 311-31)

The stochastic modelling process outlined in section 4 can include fixed-term assets such as bonds (either conventional or index-linked) that may be held to maturity. This permits the process to achieve duration matching (in either nominal or real terms) for fixed-term liabilities. Thus asset and liability modelling is consistent not only with portfolio theory but also with the theory of the term structure of interest rates (e.g. Cox, Ingersoll & Ross, 1985; Vasicek, 1977), with the advantage that the stochastic process followed by the yield curve is not unnecessarily constrained by the requirements of mathematical tractability. Maitland (2001) has shown how the principal components of the South African yield curve may be derived. There is a need to include those principal components in ALMs so that the entire yield curve can be parsimoniously modelled.

A valuable further development would be to define the utility of the surplus as a stochastic process and to solve the portfolio selection problem so as to maximise the drift of that process.

Both portfolio theory and the CAPM assume that the multiple distribution function of the values of the securities in the opportunity set is elliptical. Thus a horizontal section of the joint probability density function of any two such values is an ellipse. A multivariate normal distribution is an example of such a distribution function. If the values are not so distributed, these theories cannot be applied. For risk-averse investors, particular care needs to be taken with regard to the tails of the joint distributions. Correlation coefficients, derived as they are from the entire population or sample, may be grossly misleading indicators of the interdependence of the values. Extreme value theory may be used to model the tails of the distributions. Although the central limit theorem for extreme values converges slowly, evidence suggests that extreme value theory gives a better basis for modelling the values of

securities than conventional modelling techniques. (Embrechts, Klüppelberg & Mikosch, 1997)

Other pricing models, such as arbitrage pricing theory (Elton & Gruber, 1995: 368-97), have less relevance to the portfolio selection problem. Although they may be used for pricing liabilities, they cannot be used to determine an appropriate asset mix. Factor models can, however, be used to analyse exposures to various factors (e.g. Panjer, 1998: 410-1). Once again, this should be net of liabilities to have any meaning.

6. WHAT VALUE?

The 'surplus' of a financial institution is defined above as the excess of the value of its assets over the value of its liabilities. This begs the question how these values will be determined.

The guidance notes of the Actuarial Society of South Africa require the actuary to calculate the value of the liabilities of a life office or retirement fund on a realistic basis—'prudently realistic' in the former case and 'based on the actuary's best judgement of future events' in the latter. (Actuarial Society of South Africa, 2000) In both cases, assets must be valued on a basis consistent with the liabilities.

In practice, life offices value their assets at market value (or equivalent) and their liabilities on a consistent basis. In addition, life offices make undisclosed allowances for future profits. Retirement fund practice varies, but it is common to value liabilities on a stable value from one valuation to the next and to value assets on a consistent basis, i.e. as a discounted value of future proceeds from investments, generally below market value. The investments so valued may be the actual investments held or a notional portfolio, of equal market value, designed to match in some sense the liabilities of the fund. As Exley, Mehta & Smith (1997: 863) observe, this is not, in the strict sense of the word a 'valuation' but more properly a 'budget'. They point out that, if the notional portfolio hedges the liabilities, then the actuarial funding approach will give a realistic value in the sense that it is consistent with the market. But if, as is generally the case, notional portfolios are otherwise determined, actuarial claims to the realism of their valuation results are dubious.

It would be possible to use an ALM to develop a model for pricing risk in the same way that the CAPM uses Markowitz portfolio theory. For example, if the ALM produces normally

distributed single-period returns for the asset categories modelled, then assuming a market that maintains equilibrium and homogeneous expectations corresponding to the ALM used, the covariance matrix of those returns can be used to calculate the market price of risk over that period as in the CAPM. (Single-period returns can be used because it may be assumed that, at the end of that period, assets can be traded. The duration of long-dated assets, including equities, is reflected in the variability of their single-period returns.)

In general, it cannot be assumed that the market's expectations will correspond to those of the ALM adopted for the purposes of the management of the financial institution concerned. This could be tested by comparing the variances determined from the ALM with those implied by the prices of options on the asset categories modelled. If that test is failed, then the ALM could in principle be used for trading purposes. However, questions of credibility arise. ALMs are generally designed for long-term projections, so their use for short-term trading would be naïve. An alternative approach would be either to adjust the parameters of the ALM so that the distribution of the single-period returns correspond to those implied by the market, or to include in the ALM the values of prices or indices that reflect such implications.

In general, ALMs produce single-period returns that are not normally distributed. This means that the pricing of risk cannot generally be handled in a mean-variance framework. For the particular ALM used it would be necessary to identify risk factors (other than or in addition to the standard deviation of the return on the market portfolio) and to price them (i.e. to determine the excess expected return over the risk-free rate used in the model) as a function of those factors.

If the liability payments due at any future time are fixed in nominal terms, their market value (assuming no arbitrage) would be equal to that of a matched portfolio of default-free conventional bonds. If they are a linear function of an index (such as a price index) on which default-free index-linked bonds are available, their market value would be equal to that of a matched portfolio of default-free index-linked and conventional bonds. If they are a non-linear function of such an index (e.g. a linked benefit with a nominal guarantee), they could still be dynamically hedged with such bonds. However, in order to determine a market value, assumptions would have to be made about the volatilities of bond prices. This could either be done with reference to an ALM or (if they are available or can be created for the required expiry dates) with reference to the prices of options on bonds or on bond indices or futures.

It has been suggested to the author that, in view of the small size of the index-linked bond market in South Africa, that market cannot be used for pricing liabilities. In particular, it has been argued that, if all retirement funds were to enter the market in order to buy sufficient index-linked bonds to match their liabilities, the yields would be forced down to completely unacceptable levels. The question, of course, is: Why haven't they? And the answer is because they consider the extra expected returns on equities to be worth the mismatch risk. The fact that the market is small does not mean that it cannot be in equilibrium. And if it is in equilibrium then it can be used to price liabilities. Even for maturities at which no bonds exist, the yield curve can be used as an indication of the yields that would apply. And if they wished to do so, investment managers could create the exposures they require by buying forwards on the index. If there are no market participants willing to write such forwards, the question must be asked why trustees are willing to accept the task of meeting such liabilities and why employers are willing to underwrite them. In part, the answer is that many have stopped doing so by converting to defined-contribution retirement funds. But in respect of the rest, the question remains.

In general, the liabilities of a financial institution are not deterministically based on such indices. While other sources of risk may arguably be ignored for the purposes of benchmarking, they cannot generally be ignored for the purposes of the valuation of liabilities. Some of these risks (such as mortality risks) may be hedged or priced by means of reinsurance (Borch, 1990), or diversified out by means of pooling. Because of moral hazard, others (such as salary increases in excess of the price or wage index modelled) cannot. Exley, Mehta & Smith (*op. cit.*) attempt to use a hedging method to price such liabilities, but they do not allow for fund-specific risks that cannot be hedged. In order to use the ALM for the valuation of the latter type of risks it would be necessary to extend the ALM to include them. This would necessitate a different ALM for each financial institution. It is possible, however, that a single structure would accommodate such variations, with certain specific parameters to be estimated for each institution in respect of the factors used to model such risks, particularly since it would not be necessary to allow for any feedback from such factors. In the absence of an equilibrium market price, such a value may be calculated by recourse to expected utility theory as the price of a portfolio of assets whose proceeds have the same probability distribution (as explained in the next paragraph). The ALM may be used in order to find such a portfolio. If there is more than one such portfolio, diversification may be allowed for by

taking the portfolio with the lowest price. If there is none it would be necessary either to extend the ALM or to use approximations.

Strictly speaking, expected utility theory would imply that, in the absence of a market, a risky prospect could be valued at its certainty equivalent on the basis of the agent's utility function. But there is no guarantee that a counterparty would exist with a favourable certainty equivalent. A sufficient condition is that the probability distribution of the proceeds of the market assets is equal to that of the liability risk concerned. It should be noted that, if the liabilities have been calculated on the basis of expected values, the presence of such risks would generally tend to reduce the value of the liabilities. Additional capital may be required to accommodate that risk (and indeed the capital required may be greater than the reduction in the liabilities), but that is a separate issue, which is discussed below.

In the case of a retirement fund, another risk that cannot be insured or diversified out is the risk of the employer's insolvency. The liabilities under these circumstances may be limited to the value of the assets. If the assets are matched to the liabilities and are greater in value, this risk will not arise unless the fund is also exposed to other risks such as those discussed in the previous two paragraphs. To the extent that the trustees are able to avoid unnecessary risks by matching the assets to the liabilities, by insisting on the elimination of any shortfall as at the valuation date and by insuring or diversifying other risks, the value of the liabilities at that date should not be reduced because of the risk of the employer's insolvency. Beyond that extent, this risk can, however, be hedged, and it can therefore be valued by means of the ALM using numerical option-pricing methods. Similarly, in the case of an insurer, the value of the liabilities on the insolvency of the insurer is limited to the value of the assets and the corresponding risk can be similarly valued.

In terms of IAS 19 of the International Accounting Standards Board (IASB, 2002) the liabilities of a retirement fund must be valued, for the purposes of the accounts of employers, at a valuation rate of interest equal to the rate on high-quality corporate bonds at the valuation date. For a fund with matched liabilities, this results in an understatement, whereas for funds that cannot fully manage their risks by matching, hedging, reinsurance or diversification, it may result in an overstatement. For the latter funds, the capital requirements will be higher, as discussed below.

From the above discussion it is evident that, for particular values of marketable assets at a particular date, and for a particular ALM, the liabilities of a financial institution may be uniquely valued on a basis consistent with those values. This means that, if the market is in equilibrium and an appropriate ALM has been selected, there is no need for actuarial judgement in the setting of a valuation rate of interest, or an assumed inflation rate, for a particular liability. The only places where professional judgement may be required are therefore in the determination of reasonable benefit expectations and the selection of an ALM. And the only justifiable reason for a difference between a realistic actuarial value of the liabilities as reported to the trustees and that in the accounts of the employer is a difference of opinion between the trustees and the employer with regard to the determination of reasonable benefit expectations or the selection of an ALM.

It should also be noted that the rate of investment return effectively assumed for the valuation of the liabilities is independent of the assets actually held by the fund. Thus, by investing in assets whose expected return is higher than those of matched assets, the trustees would not reduce the value of the liabilities.

It may be appropriate to use ALM methods to determine not only benchmark portfolios and liability values, but also allowances for prudence or capital requirements. This approach, which was considered in Wilkie (1985), would give a far more quantifiable basis for the determination of such values.

Capital requirements may be based on value at risk (Jorion, 1997) determined by means of an ALM or on a risk-theoretic approach using an ALM. In the case of a retirement fund, the employer's goodwill towards and interest in the fund may take the place of capital requirements. But the trustees should ensure that the capacity of the employer to meet those requirements is adequate. The employer itself may need to increase its capital for that purpose.

The question what constitutes an appropriate method for the valuation of the liabilities of a retirement fund—i.e. what allowance should be made for future service, future salary increases etc.— is largely outside the scope of this paper. Suffice it to say that, for the purposes of financial reporting, accounting principles require that, in general, going concerns should be treated as such. Thus, even for the purposes of the employer's accounts, future

service should be considered. For the fund's accounts this is even more important. This does not necessarily mean that future accruals and contributions need to be allowed for, but it does mean that the possibility that active members will remain in service must be allowed for. This principle is more widely recognised in South Africa than in the UK, and is implicit in professional Guidance Note 201 of the Actuarial Society of South Africa (2000). Furthermore, in determining the value of liabilities, allowance must be made not only for contractual liabilities but also for reasonable benefit expectations in respect of service up to the valuation date.

In a defined-contribution retirement fund, ALM methods may be used to target defined benefits. Besides the portfolio allocations, the additional contributions required to meet the defined benefit would then constitute an additional decision variable.

7. AGENTS, PRINCIPALS AND TRUSTEES

One objection to the above analysis could be that the liabilities of financial institutions should be disregarded on the grounds that they are merely intermediaries : that, if any liabilities should be considered, it is those of the beneficiaries of the institutions concerned. In South Africa, where the bulk of internally held marketable securities are held by financial institutions, this question is critical. This argument follows the fundamentals of financial economics, whose theories are developed with reference to the intertemporal utility function of an individual consumer. The equilibrium of markets is expressed in terms of production paths and consumption streams, without reference to intermediaries (e.g. Mas-Colell, Whinston & Green, 1995: 732-77). There is some merit in this argument, but it disregards both the principal-agent problem, which is well developed in the recent literature of economics, and the role of trusteeship that must be undertaken by financial institutions (see footnote 1 above). In this section, consideration is given to both of these effects.

The role of a financial institution is to pool and manage risk and to balance equitably the interests of stakeholders. The decision-making functions associated with that role are delegated to trustees, who are advised by an actuary.

The principal-agent problem relates to the fact that the prospective beneficiaries (the principals) have entrusted certain functions to the trustees (the agents), who rely on actuaries and investment managers in making decisions with respect to a portion of their wealth. This

results in information asymmetries and moral hazard, which in turn result in different investment allocation decisions from those that the beneficiaries would have made. (Mas-Colell, Whinston & Green, 1995)

The information asymmetries function in both directions. Trustees do not know what each prospective beneficiary's exposure is to the investment opportunity set, nor do they know their respective intertemporal utility functions. Members do not have access to the actuarial and investment expertise that is available to the trustees due to the economies of scale, nor do they know in detail what criteria are being applied by the trustees for the purposes of investment or actuarial decision-making.

Moral hazard arises from the fact that the trustees' effort may fall short of the trust placed in them. The effects of this moral hazard are difficult to quantify. In the first place, the trustees' effort is at least partially unobservable, and secondly their incentives are difficult to measure.

The effects of the trusteeship function, as distinct from the principal-agent problem, are generally ignored in the economics literature. For example, retirement funding policy is generally regarded from the point of view of participating employers, not from that of the trustees. In fact, the trustees (or 'managers' as they are generally called) are perceived to be agents of the stakeholders (shareholders or participating employers) other than the prospective beneficiaries (e.g. Sharpe, 1976; Treynor, 1977).

Similarly, accountants have ignored the trusteeship function by setting the valuation rate of interest equal to that of high-grade corporate debentures (IASB, 2002). This assumes that trustees will deliberately avoid matching liabilities with risk-free assets, even where such matching is possible.

Borch (1990) recognises the need for protection of beneficiaries, but assigns it to government. This reduces protection to constraints, which may complicate the problem but do not change its fundamental definition from that of an agency problem.

However, the effects of trusteeship create further disparities between trustees' decision-making and that of the prospective beneficiaries. Not only must trustees act with utmost good faith, they must do so with prudence. They may or may not consider it their duty to counteract

what they may perceive as the reckless conservatism of prospective beneficiaries. Trustees must balance the interests of all prospective beneficiaries and, subject to certain constraints, those of other stakeholders such as shareholders and participating employers. The rules of the fund, or the provisions of the contracts, governing the payment of contributions and benefits, together with the reasonable benefit expectations of beneficiaries, act as a constraint. Finally, as illustrated by Arrow's (1963) impossibility theorem, there are substantial difficulties in aggregating the preferences of a group of decision-makers (such as trustees) so as to conform to rational and democratic criteria.

To some extent, particularly where risks are not pooled (as in the investment portion of unitised life insurance products or defined-contribution retirement funds), the trustees may provide alternative investment channels for members. In this case the benchmarks may be defined with reference to the descriptions of the channels provided. But even here it is up to the trustees to ensure that the benchmark portfolios are appropriately defined and default channels generally have to be provided for those members who prefer to leave such decisions to the trustees. This matter is discussed further in Thomson (1998a, 1998b, 2000) and Asher (1999).

To the extent that trustees fulfil the requirements of trusteeship, moral hazard is reduced and shifted away from beneficiaries and towards other stakeholders. In the past there has been a tendency for the trustees of retirement funds to be appointed by employers and to be held accountable to them more than to members. In recent years, largely in response to legislation both in South Africa and abroad, that has been changing. A major problem with the legislation is that it tends to identify each trustee as a representative either of members or of employers, thus detracting from the principle of trusteeship. On the other hand, widespread trustee training has followed these changes in South Africa.

In the past, by the same token, particularly in South Africa, actuaries formally appointed to advise the trustees of funds have in effect been appointed by employers, resulting in conflicting interests that have not always been properly handled. As a result of a growing body of case law from the Pensions Adjudicator and the courts, actuaries are now more aware of the issues involved. But employers may still control or at least strongly influence the appointment of actuaries, and actuaries compete more vigorously for appointment, so the problem persists.

Even if it were accepted that financial institutions should be ignored as mere intermediaries, however, it would still be necessary to have regard to the other assets and liabilities of individual investors. Some of these liabilities, such as loans from family members, are subject to even greater problems of counterparty risk and moral hazard than the more formal liabilities of financial institutions, and are therefore even less tradable.

Another objection to the above analysis could be couched in terms of the capital irrelevance proposition (Modigliani & Miller, 1958). This proposition claims that the value of a company is unaffected by its capital structure. If a company increases its gearing, its shareholders can decrease their gearing to compensate. By the same token, if a financial institution changes its asset mix, its stakeholders can change their own portfolios to compensate.

As is now generally recognised, however, the proposition rests on the assumption of zero transaction costs. Its usefulness is greater as a point of departure than as a statement of fact. The question it raises, therefore, is more properly to what extent the real world departs from its premises as a result of non-zero transaction costs than whether the proposition is true. As discussed above, the prospective beneficiaries of a financial institution cannot easily trade their interests in it. Furthermore, they may be generally unaware of their exposure through the institution. Thirdly, the instruments available to them may not be the same as those available to the institution, particularly on short positions. And finally, individual stakeholders incur higher transaction costs because they lack the economies of scale enjoyed by the institution. These arguments are particularly true in a third-world country such as South Africa. In addition, as pointed out by Eichberger & Harper (1997):

> When firms have limited liability, shareholders prefer the firm to be highly geared since they do not have to bear the negative cashflows in those states where the firm goes bankrupt. Consequently the Modigliani-Miller proposition breaks down. A second reason why shareholders might not be indifferent to a firm's financial policy occurs when debt and equity are taxed differently.

Liabilities for benefits are qualitatively different from capital. Not only do beneficiaries enjoy priority over other stakeholders (at least so far as their contractual rights are concerned), they also generally have benefits defined independently of the assets of the financial institution.

While debenture holders, for example, are arguably in a similar position, debenture trustees do not have control over the assets on which their rights are based unless the company is in default, in which case they would indeed be in a similar position. Furthermore, from the trustees' point of view, whereas the purpose of capital is to finance their business, that is not true of the liabilities. In fact the business of the trustees is to meet the liabilities; investment in assets is merely a means to that end. Thus, while the capital structure of a financial institution may be irrelevant to its portfolio selection problem, its liability structure is not.

It may be argued that, because the employer bears the balance of cost of the accruing liabilities in a defined-benefit retirement fund and thus bears the risks of a mismatch between its liabilities and its assets, the investment strategy of the fund should be dictated at least to some extent by the employer's interests. This argument ignores the greater risks that the beneficiaries bear and the fact that the trustees' primary role is to protect them. Nevertheless, if and to the extent that the trustees are satisfied that (as discussed in section 6) the employer's goodwill, interest and capital are adequate and they are of the view that it would be to the benefit of the fund, mismatching may be justified.

In an expected-utility framework, the only circumstances in which the liabilities would not be relevant to the portfolio selection problem would be where the trustees are universally risk-neutral. Because one of their roles is to manage risk, the implication is that, in general, risk is regarded as a problem. The only situation in which that problem would be irrelevant (and therefore the trustees would be risk neutral) is where the stakeholders are able to manage their own risks at no extra cost to the stakeholders. Under those circumstances the trusteeship function would itself be irrelevant and would merely incur unnecessary transaction costs for the stakeholders.

Thus, as Smith (1996: 1072-3) argues in the context of an insurer, the portfolio selection problem remains relevant to a financial institution. In addition, the trusteeship principle requires that the financial institution should not expect prospective beneficiaries to compensate for failure on its own part to address the portfolio selection problem in utmost good faith. Thus, in exercising their trust, the trustees of a financial institution need to invest its assets and quantify its risks with reference to its liabilities.

In this paper, no reference is made to the business risks of the financial institution. Just as liabilities need to be considered in the portfolio selection problem, so, in principle, do business risks. Whilst the principle is the same, the particulars are in some respects different. For fuller treatment the reader is referred to Mehta (1992).

8. CONCLUDING REMARKS

It is clear from the above outline not only that it is incorrect to consider investment strategy or performance without reference to the investor's liabilities, but also that, even where the liabilities are untradable, there is now a well-developed body of theory that may be applied in determining such strategy and measuring such performance with reference to those liabilities. This is particularly important where the return on one or more of the asset categories under consideration is interdependent with the growth of the investor's liabilities. In the case of Patrick & Ward (1996) and Swart (1999) consideration was given to the benefits of international diversification to South African investors without reference to their liabilities. It is likely that such assets are correlated, at least to some extent, to the typical liabilities of such investors. The theory of purchasing power parity (e.g. Samuelson & Nordhaus, 1985: 875-6) suggests that, when one currency weakens against another, the prices of goods and services in that country will tend to rise relative to the other. Thus, while an investment in a strong currency will not provide as good a hedge against inflation as inflation-protected investments in the domestic currency, it may be expected that, in the longer term, they will provide a partial hedge against inflation of the investor's liabilities in the weaker currency. This effect could have been readily tested by means of an asset and liability model.

In this paper the following theses are defended:

- that, in exercising their trust, the trustees of a financial institution need to invest its assets and quantify its risks with reference to its liabilities;
- that for that purpose, the value of the liabilities, the capital required and the quantification of the risks may be determined from market prices of assets, together with an ALM; and
- that the investments of the financial institution should be managed with reference to a benchmark portfolio selected by the trustees from an efficient frontier in risk-reward space, where risk and reward are appropriately defined and determined by the ALM.

From the discussion in this paper it is clear that further research is required in certain areas, notably:

- the development of measures of capital adequacy using ALMs;
- the development of ALMs for financial institutions to incorporate recent developments in modelling techniques;
- the development of measures of risk incorporating the liabilities of the financial institution;
- an empirical study of trustees' attitudes towards risk;
- the incorporation of the principal components of the yield curve into ALMs;
- the solution of the portfolio selection problem in continuous time with allowance for liabilities;
- the application of extreme value theory to problems involving assets and liabilities;
- the use of ALMs for the pricing of risk;
- the incorporation into ALMs of market data, such as implied volatilities, that can be used as state variables for the simulation of subsequent values of the variables modelled;
- the modelling of retirement fund risks associated with real salary increases, distinguishing between diversifiable risk and systematic risk;
- the modelling of retirement fund risks associated with the insolvency of the employer;
- the extent to which moral hazard is reduced and shifted away from beneficiaries and towards other stakeholders through the trusteeship function;
- the extent and effect of conflicting interests of actuaries advising financial institutions; and
- a reconsideration of the benefits of international diversification to South African investors with reference to their liabilities.

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ACKNOWLEDGEMENTS

The author acknowledges with thanks the help of Professor A Asher and Dr Z Khorasanee with regard to certain aspects of this paper, as well as contributors to the discussion of an earlier draft at a seminar held for that purpose, and the particularly helpful comments of an anonymous referee.