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A MATHEMATICAL APPROACH TO INCREASING THE LONG-TERM WEALTH OF AN AGRICULTURAL ENTERPRISE[1]

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ABSTRACT
This study focuses on developing an agricultural investment model based upon proven financial investment portfolio techniques. The model can be used as a tool to diversify agricultural risk over the long-term by optimising the proportion of land allocated to each of the agricultural products, resulting in increased value of the agricultural enterprise. Sensitivity analysis allows the strategist to understand the impact that future prices, gross margins and land availability may have on the long-term sustainability of the farming enterprise.

KEYWORDS
Agriculture, diversification, efficient frontier, expected return, long-term, risk, portfolio model, gross margin, quadratic programming

INTRODUCTION

The traditional role of management is to increase the value of a firm. In order to successfully increase value, management should focus on increasing profits by operating more efficiently, driving down costs and increasing revenue. Analysis may lead to certain products being discontinued and others adopted in order to maximise the firm's profits. The volatility of demand (and resulting price per unit) of these products may impose certain additional risks on the business due to the uncertainty associated with these future prices. These risks usually

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[1] This paper is based on the first author's winning entry in the 2001 ORSSA Student Project Competition. His research project was entitled "Minimising Long-Term Agricultural Price Risk: A Quadratic Programming Model based on the Markowitz Mean-Variance Approach to risk Minimisation".

transform into higher costs of capital, as stakeholders demand higher rates of return or interest to offset the higher risk that their investments may be exposed to. By attempting to diversify the product portfolio, the firm may be able to reduce the portfolio risk.

Given that most agricultural products such as wheat, maize, coffee and meat are commodities, and thus highly influenced by demand and supply (which leads to volatile price adjustments in free market economies), it is important for the producers of these products to choose the correct product range and weightings in order to maximise the overall profitability of the business and minimise the risks associated with potential price movements over the long term.

In this paper we develop an optimisation model to advise managements of agribusinesses on optimal agricultural product portfolios and their weightings. In the next section we develop the model. Thereafter we describe how this model may be implemented on a spreadsheet. An implementation case study is then discussed, followed by some observations on real world applications of the tool.

THE MODEL

In general, the current value of a business is the present value of the sum of its future cash flows (adjusted for capital expenditure). The *Free Cash Flow to Firm* approach is a popular model for valuing a business and forms the basis of more complex models used by many financial institutions (see, for example Damodaran (1996: 242)). Using this model

$$\text{Value of Firm} = \sum_{t=1}^{t=\infty} \frac{\text{FCFF}_t}{(1+\text{WACC})^t}$$

where

FCFF_t = free cash flow to the firm in year t

WACC = weighted average cost of capital.

With reference to the above formula, it can be seen that value can be increased by either decreasing the weighted average cost of capital of the firm, or by increasing the free cash flow to the firm. The most desirable effect would then be a combination of increased free cash flow through growing profits, whilst at the same time driving down the weighted average cost of capital by reducing various forms of risk within the business.

In March 1952, Harry M. Markowitz published his now-famous paper in the *Journal of Finance* entitled "*Portfolio Selection*" (1952:77-91). In it, he demonstrated how to reduce the standard deviation of returns on asset portfolios (i.e. portfolio risk) by selecting assets that do not move in exactly the same way as each other. In the same article he laid down some basic principles for establishing an advantageous relationship between risk and return.

Although this model was developed and applied with the optimisation of financial security portfolios in mind, it is possible to apply this theory to a different field, that of optimising the resources utilised for producing agricultural products. Depending on the risk profile of the agricultural firm and the resulting strategies adopted by it, the quadratic programming model developed here may be used to minimise risk and increase free cash flow to the agricultural enterprise, and in so doing increase the potential value of the business.

The Markowitz model, adapted for use in the agricultural industry, can be written as:

Minimise Risk $$\sum_{i=1}^{n}\sum_{j=1}^{n} P_i P_j \sigma_i \sigma_j \rho_{ij}$$

Subject to:

Required adjusted price increase $$\sum_{i=1}^{n} g_i P_i r_i \geq R_a$$

Productive land constraint $$\sum_{i=1}^{n} P_i = 1$$

Category A constraint $$\sum_{a=1}^{x} P_a \leq C_1$$

Category B constraint $$\sum_{b=1}^{y} P_b \leq C_2$$

Category C constraint $$\sum_{c=1}^{z} P_c \leq C_3$$

where:

P_i = proportion of land allocated to agricultural product *i*

P_a = proportion of land allocated to category A products

P_b = proportion of land allocated to category B products

P_c = proportion of land allocated to category C products

n = total number of products under consideration.

x = total number of category A products under consideration where $x \leq n$

y = total number of category B products under consideration where $y \leq n$

z = total number of category C products under consideration where $z \leq n$

σ_i = standard deviation of price returns of product i

r_i = expected price growth of product i

g_i = expected gross margin of product i

R_a = required adjusted return of the agricultural portfolio

C_1 = portion of land available for cultivation of category A products

C_2 = portion of land available for cultivation of category B products

C_3 = portion of land available for cultivation of category C products

ρ_{ij} = correlation of price returns between agricultural products i and j.

When comparing the assets and their returns of the security markets with the returns and assets of a typical farm, some fundamental differences emerge, and thus certain assumptions need to be made before the model may be used with any degree of success or reliability.

The returns on a security may be written as $R_s = f(\Delta P, Q_0, c, t)$ where R_s is the return generated by changes in the security price[2] ΔP over a certain time period t by a certain quantity of securities Q_0 initially invested. The small costs involved in commissions and transactional costs c may be considered as insignificant. The only significant variable in the function is ΔP. Thus for all intents and purposes, we could re-write the above equation as $R_s \approx f(\Delta P)$.

Let us compare this to the return generated by an agricultural product: $R_p = f(\Delta P, \Delta Q, \Delta VC, FC, t)$ where the return generated by the agricultural product R_p is a function of the change in product price between the time of planting and harvesting ΔP, and ΔQ represents the yield (quantity) of the product (e.g. tons per hectare). The variable costs ΔVC and fixed costs FC of producing this product must also be considered. The gross margin GM of each product is defined as $(P-VC)/P$. The time period between planting and harvesting is denoted by t. By assuming that FC is constant we could re-write the above equation as $R_p \approx f(\Delta P, \Delta Q, \Delta VC)$.

If we assume that the yield of each product remains fairly constant over time, and recognising that gross margin is a function of both price and variable costs, the return of the agricultural product may be written as $R_p \approx f(\Delta P, \Delta GM)$.

Thus in the case of equity securities the return generated by assets is a function of a change in price only (ignoring the effects of dividend yields), whilst the return generated by an agricultural product is a function of a change in price *and* the marginal contribution of the products produced.

Should the agricultural model only take the changes in prices ΔP into account, we would be solving only part of the problem. The model would prove most accurate if the historical changes in *contribution margin* were to be used. But if, over the long-term, the yields and variable costs are known and considered to be fairly consistent (i.e. less volatile relative to expected change in prices), and if this is consistent over the entire range of products, then the expected change in price would be the major driver which would result in the contribution being more volatile (i.e. the volatility of the contribution over time would be most dependant on the volatility of the price). This future expected price growth of the agricultural product must be weighted by its gross margin percentage to avoid biased allocations of land to products with potential for high price growth, but with low gross margins. Given that the model is an attempt to minimise risk over the long term, the following equation must then hold: $R_p \approx E(\text{Contribution}) \approx E(P.GM)$.

Note that this model is best suited for long-term strategic decision-making where land utilisation is the major resource employed in producing the agricultural products. The model could be used with some degree of success in the short-term if the costs[3] of switching between divesting product and investing product is low. For example, the switching costs between barley and wheat would be considerably lower than the switching costs between an apple orchard and a pear orchard.

Referring to the constraints in the adapted Markowitz model shown above, the historical change in price of each agricultural product under consideration is weighted by its gross margin contribution, resulting in an adjusted price increase metric. The model also allows for

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[2] We assume that the dividend yield has been built into the share price in the case of equity securities.

constraining several categories of products (we have indicated three such categories). For example, an enterprise may have fertile land close to irrigation facilities which would be suited to crops requiring a high degree of irrigation. Fertile land with no access to irrigation facilities, on the other hand, may be restricted to certain crops, which may thrive under these conditions (*dry land* crops). Specific soil composition may also limit land suited for *orchards*, for example. The balance of productive land may be assumed to be available for products not grouped into a specific category, for example livestock such as cattle and sheep. The *productive land* constraint does not include land occupied by non-operating assets such as residential housing, roads, sheds etc.

IMPLEMENTING THE MODEL IN A SPREADSHEET

A simple Microsoft Excel spreadsheet can be used to apply the model to real world applications. Some VBA (Visual Basic for Applications) coding is used to streamline the macro and perform multiple iterations through the *Solver* Add-in (which is part of the Microsoft Excel package). For example, below is an extract of code that is used to plot the efficient frontier.

```
'Solve Frontier values
    SolverOk SetCell:="$O$4", MaxMinVal:=2, ValueOf:="0", ByChange:=Range("C2:C101")
    For i = 1 To a
        Range("R3").Value = e + (i * f)
        SolverSolve UserFinish = False
        Range(Cells(i + 19, 15), Cells(i + 19, 15)).Value = Range("O4").Value 'Risk
        Range(Cells(i + 19, 16), Cells(i + 19, 16)).Value = Range("O3").Value 'Return
    Next i
```

Initially, data is entered into a data template, which is divided into three sections (see Appendix 1 for a snapshot view of an extract of the template). The first section of the template requires the user to enter the periodic change in price of each product under review. The monthly product price changes (or monthly returns) are calculated by using the following formula:

$$\text{Monthly Return}_{i+1} = \frac{\text{Price in Month}_{i+1} - \text{Price in Month}_{i}}{\text{Price in Month}_{i}}$$

[3] These costs would include the opportunity cost of capital, as it may take time to generate income.

An extract showing monthly returns can be seen in Figure 1 (note that areas shown with a shaded background require input from the user).

Maize	Wheat	Lucerne	Cattle	Sheep	Pigs	Barley	Sugar
0.0%	0.0%	0.0%	0.7%	0.0%	-0.7%	0.0%	0.0%
0.0%	0.0%	0.0%	0.0%	-9.5%	-13.8%	0.0%	0.0%
0.0%	0.0%	17.6%	-0.4%	-5.6%	-7.8%	0.0%	0.0%
54.7%	0.0%	0.0%	1.2%	-4.3%	0.6%	0.0%	-1.8%
0.0%	0.0%	0.0%	-4.2%	-2.1%	-1.9%	0.0%	0.0%
0.0%	0.0%	8.8%	-9.4%	-1.6%	-3.0%	0.0%	0.0%
0.0%	0.0%	0.0%	-2.7%	6.9%	3.2%	0.0%	0.0%
0.0%	0.0%	0.0%	2.5%	-4.1%	5.9%	0.0%	0.0%
0.0%	0.0%	-23.5%	-2.5%	5.0%	13.3%	0.0%	0.0%
0.0%	4.3%	0.0%	6.1%	0.9%	-2.6%	7.2%	0.0%
0.0%	0.0%	0.0%	8.9%	13.8%	11.5%	0.0%	0.0%

Figure 1 - *Data template: monthly returns for agricultural products*

Monthly prices for this study were obtained from the South African Department of Agriculture and can be manipulated into the correct format with the help of a spreadsheet. Appendix 2 contains an extract of original product prices.

The second section of the data template allows the user to group certain products into different categories, which will form constraints within the model. The expected annual price growth and gross margin of each potential product are also entered into this template. Historical average results for the expected price growth and gross margins may be used, but it is advised that these be adjusted with future performance in mind; for example the potential performance of wheat may be adversely affected by a move to wheat-free products by consumers. In this case the future price growth of wheat should be deflated by a suitable percentage. Any potential future variable costs associated with products under review should also be accounted for by adjusting the gross margin percentages.

A column named "*Current Land Utilization*" requires the user to enter the area of land currently occupied by each specific product. The units must be consistent throughout the model; in this case hectares are used. An example of this section of the data template can be seen in Figure 2.

	Products	Enter Category	Expected Price Growth	Gross Margin	Current Land Utilization [Ha]
1	Maize	A	7.7%	5.0%	-
2	Wheat	A	5.6%	5.0%	1,331
3	Lucerne	A	6.6%	5.0%	-
4	Cattle		0.7%	13.0%	5,790
5	Sheep		2.9%	13.0%	3,083
6	Pigs		2.0%	13.0%	-
7	Barley	A	3.3%	5.0%	1,993
8	Sugar	A	4.3%	34.0%	7,257
9	Apples	B	6.2%	1.0%	145
10	Bananas	C	4.8%	23.0%	400
11	Pears	B	15.6%	1.0%	21
12	Avocados	C	10.4%	23.0%	13
13	Citrus	B	6.7%	1.0%	340

Figure 2 - *Data template: expected price growth, gross margins and current land utilisation for agricultural products*

The third section of the template allows the user to enter labels for the three category constraints and select whether the current land utilization should be less than or equal, or equal to the *Land Available Constraint* specific to that category. This is shown in Figure 3. The *Required Adjusted Return* is the minimum weighted sum of the returns on all the products under review required by the agribusiness.

	Description of Category	Current Utilization [Ha]		Land Available Constraint [Ha]
A	Irrigated Crops	10,581	≤	15,000
B	Drip Irrigated Orchards	506	≤	1,000
C	Dry Land Plantations	413	≤	1,000
	Balance	8,873		3,373
		Required Adjusted Return	≥	0.66%
		Current Adjusted Return		0.66%
		Total Available Productive Land		20,373
		Cycle Time (years)		10

Figure 3 - *Category and land available constraints*

The *Balance* quantities (current and available) are the difference between the *Total Available Productive Land* and the sum of the three category totals. Should this balance be negative, the spreadsheet will prompt the user to correct the mistake and will not attempt to solve the model until all the entries have been checked for validity.

The *Cycle Time* is used to calculate a proxy value of the enterprise by using the Free Cash Flow to Firm formula. A cycle may coincide with the productive life expectancy of a fruit orchard[4], for example.

The user also has the option of choosing to solve and plot the current and optimal positions of the portfolio of agricultural products on an efficient frontier.

The macro generates a new sheet called "*Model*"[5], which contains the optimal solution and the efficient frontier, if this has been selected. Appendix 3 contains a snapshot of the output sheet. The model is dynamic in the sense that the user has the ability to change values in the template and see the changes to the optimal solution of the model. "What-if" analysis can be exercised this way. The model output will be described in more detail in the next section.

ILLUSTRATIVE RESULTS

We will illustrate the use of the model by introducing an example based on real price information gathered from the South African Department of Agriculture over a sixty-month period, and information submitted from a leading agricultural enterprise (which is listed on the Johannesburg Securities Exchange). A variety of scenarios have been compiled in order to demonstrate various rational and irrational long-term agribusiness strategies.

We will start with a base case where an established agricultural enterprise would like to map its risk-return position relative to the optimal mix of products in order to minimise the portfolio risk, given the current category constraints.

The established products as well as those under review, together with their expected returns and gross margins are shown in Figure 4.

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[4] Most long-term agricultural products have limited "productive life spans" and have to be replaced periodically. This model helps assess the optimal product for the next long-term cycle.

[5] The macro will delete any sheets with the name "Model" already contained within the spreadsheet, before generating a new solution.

	Products	Enter Category	Expected Price Growth	Gross Margin	Current Land Utilization [Ha]
1	Maize	A	7.7%	5.0%	-
2	Wheat	A	5.6%	5.0%	1,331
3	Lucerne	A	6.6%	5.0%	-
4	Cattle		0.7%	13.0%	5,790
5	Sheep		2.9%	13.0%	3,083
6	Pigs		2.0%	13.0%	-
7	Barley	A	3.3%	5.0%	1,993
8	Sugar	A	4.3%	34.0%	7,257
9	Apples	B	6.2%	1.0%	145
10	Bananas	C	4.8%	23.0%	400
11	Pears	B	15.6%	1.0%	21
12	Avocados	C	10.4%	23.0%	13
13	Citrus	B	6.7%	1.0%	340

Figure 4 - *Agricultural products considered, with expected returns and gross margins*

The products have been grouped into specific categories, which are constrained due to the area of suitable land available for producing that particular product. The cycle time of ten years has also been entered into the template as shown in Figure 5.

	Description of Category	Current Utilization [Ha]		Land Available Constraint [Ha]
A	Irrigated Crops	10,581	≤	15,000
B	Drip Irrigated Orchards	506	≤	1,000
C	Dry Land Plantations	413	≤	1,000
	Balance	8,873		3,373
		Required Adjusted Return	≥	0.66%
		Current Adjusted Return		0.66%
		Total Available Productive Land		20,373
		Cycle Time (years)		10

Figure 5 - *Input to category and land available constraints*

With reference to the category constraints, it can be seen that the current land utilization of 10 581 Ha of *Irrigated Crops* is less than the potential 15 000 Ha available. The same is also true for the other two category constraints, with the balance of land of 8 873 Ha currently allocated to livestock.

The optimised model which minimises portfolio risk (the results are shown in Figure 6), shows that it is possible to generate a reduction in risk of 29.9% with a 15.8% gain in portfolio-adjusted return. Using a proxy for firm value based on the Free Cash Flow to Firm model (see Appendix 4 for details of the proxy value calculation), a 20.5% improvement in firm value can be attained, the main driver of this being the 16% reduction in portfolio risk. It should also be noted that all three of the category constraints are binding, indicating that further value might be gained should the firm invest in more production facilities suitable for crops rather than livestock.

Model Results	**Optimal Case**	**Current Case**		**Para-meters**	**Change on Current**
Portfolio Adjusted Return	0.77%	0.66%	≥	0.66%	15.75%
Portfolio Risk	4.35%	6.20%			29.85%
Irrigated Crops	15,000	10,581	≤	15,000	41.76%
Drip Irrigated Orchards	1,000	506	≤	1,000	97.63%
Dry Land Plantations	1,000	413	≤	1,000	142.13%
Balance	3,373	8,873		3,373	61.99%
Total Productive Land	**20,373**	**20,373**		**20,373**	
Proxy Value					
Term	10				
Cash Flow Factor	1.079452	1.06832			1.04%
Risk Factor	1.530255	1.82419			16.11%
Proxy Value	0.705406	0.58564			20.45%

Figure 6 - *Optimised portfolio results*

The risk-return efficient frontier is displayed in Figure 7. The optimal solution (Position C) forms the base of the efficient frontier, which stretches up and to the right. Any combination of adjusted return and risk on the efficient frontier would be a rational and in a sense optimal one. Choosing a strategy that would result in the firm aiming for Postion B would be an irrational one as a higher level of adjusted return may be realised from Position D for the same level of portfolio risk. Adopting Position C may be referred to as a passive strategy, in investment terms.

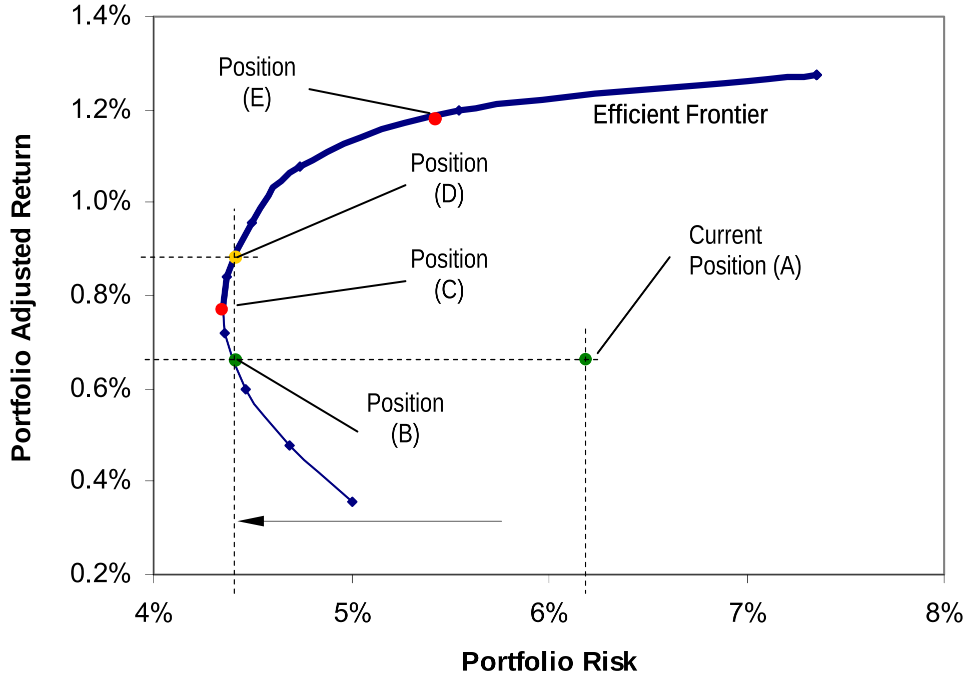

Figure 7 - *Efficient frontier*

The current portfolio of agricultural products yields an adjusted return and risk shown as Position A. In order for the firm to move from Position A to Position C it would have to reduce its investment in wheat, cattle, sheep, apples, bananas and avocados and increase its investment in maize, lucerne, barley, sugar and citrus. The changes in products between Points A and C are displayed graphically in
Figure **8**. This would mean an increase in the proxy value of 21% (driven mainly by a reduction in the risk factor) and of portfolio adjusted return of 16%, given a reduction of portfolio risk of 30%.

With reference to
Figure **8**, the practical implications of reducing land available for sheep and increasing land available for sugar plantations would entail the ploughing of land, planting of sugar cane and the installation of irrigation infrastructure. The proceeds from the sale of the sheep may contribute towards the development of the sugar plantation[6]. In essence, sheep (which falls into the 'balance of productive land' category) currently occupy land available for irrigation (sugar forming part of this category).

[6] This would form part of the feasibility study of replacing sheep with sugar and may include immediate financing and resource constraints. This short term detailed analysis is important, but does not form part of the discussion of this paper, which is focused more on the long-term implications.

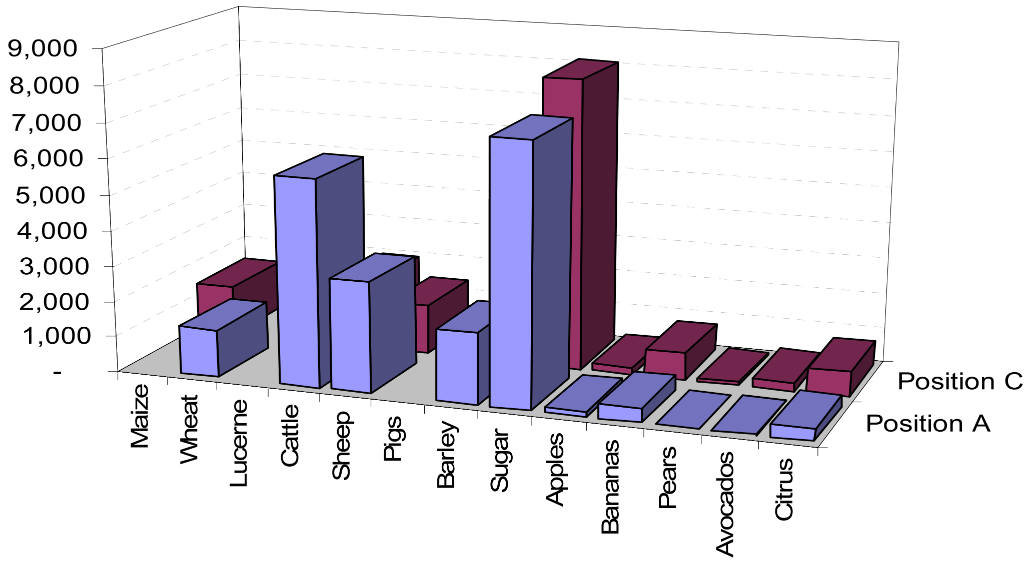

Figure 8 - Land allocations for current portfolio and the minimum risk portfolio

Land allocations amongst all products for Positions A to E on the efficient frontier are given in Appendix 5. Assume that the firm decides to take a more aggressive approach and makes a 10-year strategic decision to position itself on the efficient frontier at Position D in Figure 7. This would mean striving for a higher adjusted return by accepting a higher degree of risk. The results of this scenario appear in Figure 9.

Model Results	**Optimal Case**	**Current Case**		**Para-meters**	**Change on Current**
Portfolio Adjusted Return	0.82%	0.66%	≥	0.66%	24.27%
Portfolio Risk	4.39%	6.20%		4.39%	29.10%
Irrigated Crops	15,000	10,581	≤	15,000	41.76%
Drip Irrigated Orchards	1,000	506	≤	1,000	97.63%
Dry Land Plantations	1,000	413	≤	1,000	142.13%
Balance	3,373	8,873		3,373	61.99%
Total Productive Land	**20,373**	**20,373**		**20,373**	
Proxy Value					
Term	10				
Cash Flow Factor	1.085521	1.06832			1.61%
Risk Factor	1.537146	1.82419			15.74%
Proxy Value	0.706193	0.58564			20.59%

Figure 9 - *Optimised solution at Position D*

As may be expected there is a significant increase in portfolio-adjusted return (24.3%) from the status quo portfolio, with a reduction of 29.1% in portfolio risk relative to the current position.

Having discussed the situation where a firm is constrained by its land availability and would like to optimise its strategic (but passive) position, we will now consider the scenario where the firm decides to improve its long-term position by acquiring more productive land. Assume that on analysing the previous scenarios, the agribusiness decided to acquire an additional 5,000 Ha of fields fit for growing crops requiring irrigation and would like to view the effect that this acquisition would have on the business over the cycle period of 10 years. The expected change in risk/return space is depicted in Figure 10.

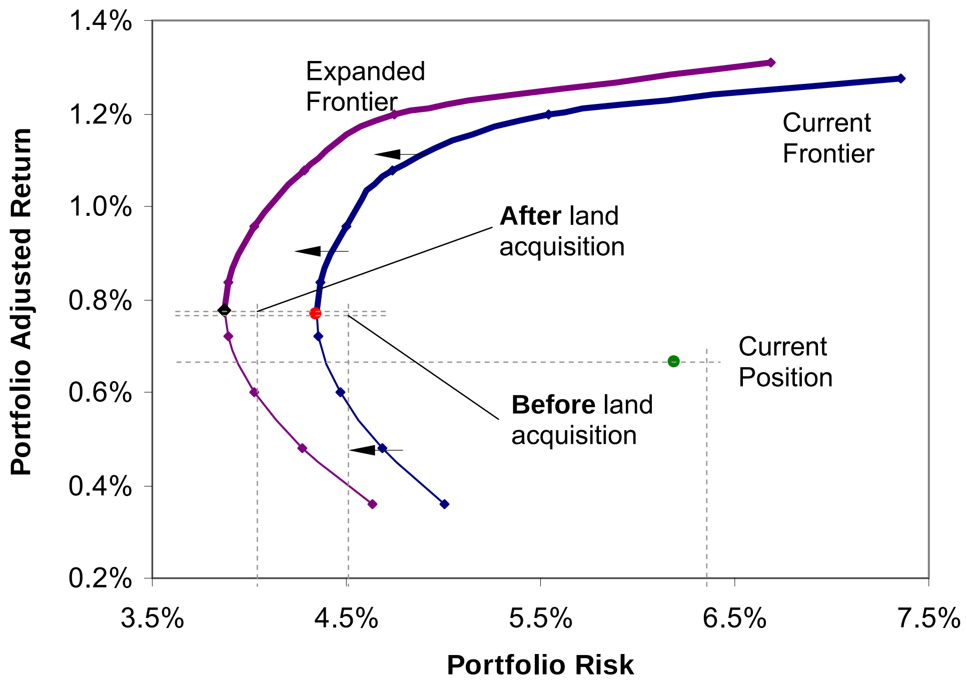

Figure 10 - *Shift of the efficient frontier*

From Figure 10 it can be seen that the efficient frontier has shifted left and slightly upwards compared to the 'current' frontier. In effect, by acquiring and developing an additional 5,000 Ha of land available for irrigation, the agribusiness has been able to reduce its portfolio risk a further 11% with a slight gain of 1.32% on adjusted return, this effectively leading to an additional increase of 4.75% of the proxy value (see Appendix 6 for a summary of these values). Figure 11 displays the optimal area allocated to each product under the base scenario, and the case of the area expanded by 5,000 Ha. Figure 11 displays the optimal area allocated to each product under the base scenario, and the case of the area expanded by 5,000 Ha.

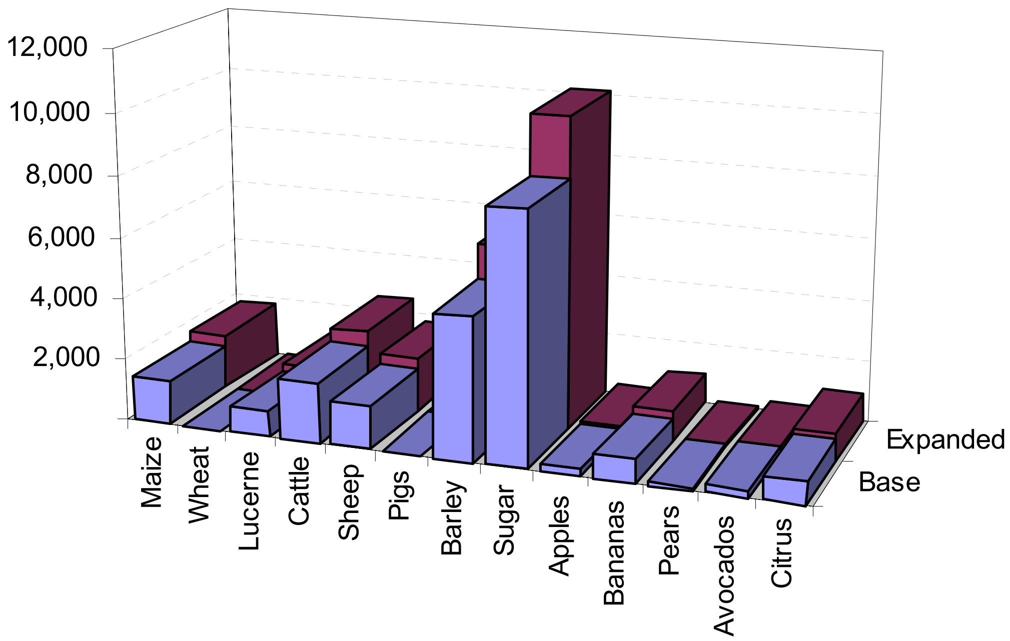

Figure 11- *Land allocations for expanded and base scenarios*

Barley and Sugar take up more than 90% of the new 5,000 Ha expansion, occupying respectively 26.3% and 41.7% of the total land available in the expanded enterprise.

PRACTICAL USE OF THE MODEL

It should be recognised that some products (such as fruit, coffee etc) have a considerable lead-time to yield. Thus year-to-year switching by an agribusiness would usually not be practical as the switching costs would be high (e.g. capital costs associated with replanting a plantation, vineyard or orchard; a number of years of no yield before maturity of the crop etc). This is in sharp contrast to the case of financial securities, where switching costs are relatively low (small commissions and transaction costs), and yield is continuous.

The model presented here thus focuses on long-term agricultural investment through development of a new piece of land or through a one-off restructuring of existing farming assets. The potential products that would be considered for analysis would be those that are suited to the climatic conditions and soil types available on the relevant agricultural land, and grouped into relevant categories (for example wheat, oats and barley may form a category called “winter grains”, given that they thrive in similar climatic conditions and soil types).

This approach may offer real practical advantages in decision support. A South African agribusiness indicated that it intends to test the model on certain long-term, but well-defined decisions.

CONCLUSIONS

By using this model and running various scenarios, it is possible for an agribusiness to quantify its strategic position in risk, return and value terms. This allows the decision makers to develop the enterprise by aiming for a rational position along the efficient frontier and developing and acquiring productive land that optimises this position over the long term.

By embarking on a strategy which moves the enterprise’s position closer to the efficient frontier, the agribusiness is likely to reduce its adjusted portfolio risk whilst increasing return, which leads to an increase in overall value of the enterprise.

The benefits of this model lie in its ability to use the relationships between prices of agricultural commodities with each other (i.e. the magnitude of covariance), and thus the ability to reduce risk by selecting the best mix of products over the long-term, to help the user make a reasonable assessment of what the optimal mix should be from a strategic point of view.

Given the inherent nature of the various products under consideration and the way their prices move in relation with one another, it is unlikely that the covariance between them will change significantly from one long-term cycle to the other. In other words, once the optimal portfolio has been identified and initiative taken to transform the agribusiness from current status to that shown by the model to be optimal (within the bounds of the agribusiness’s risk policy), it is unlikely that the agribusiness will have to make huge capital investments every few years in order to remain optimal unless there are drastic price adjustments or changes in gross contribution margins.

In essence, this model yields the most meaningful results when new acquisitions or expansions are considered (off the base of a currently optimal product mix) or where the agribusiness has reached the end of its products' productive life cycle and is considering what the best next step over the next long-term cycle should be. The economic cost implications of switching should always be balanced against the long-term strategic advantages when the model suggests agribusiness transformation.

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APPENDICES

Appendix 1 – Extract of the template used to enter data into the model

	Products	Enter Category	Expected Price Growth	Gross Margin	Current Land Utilization [Ha]
1	Maize	A	7.7%	5.0%	
2	Wheat	A	5.6%	5.0%	1,331
3	Lucerne	B	6.6%	5.0%	
4	Cattle		0.7%	13.0%	5,790
5	Sheep		2.9%	13.0%	3,083
6	Pigs		2.0%	13.0%	
7	Barley	A	3.3%	5.0%	1,993
8	Sugar	B	4.3%	34.0%	7,257
9	Apples	C	6.2%	1.0%	145
10	Bananas	C	4.8%	23.0%	400
11	Pears	C	15.6%	1.0%	21
12	Avocados	C	10.4%	23.0%	13
13	Citrus	C	6.7%	1.0%	340
14					
15					
16					
17					
18					
19					
20					
21					
22					

	Description of Category	Current Utilization [Ha]		Land Available [Ha]
A	Irrigation Crops	3,324	≤	4,500
B	Dry Land Crops	7,257	≤	8,000
C	Orchards	919	≤	2,000
	Balance	8,873		5,873
	Required Adjusted Return		≥	1.00%
	Current Adjusted Return			0.66%
	Total Available Productive Land			20,373
	Cycle Time (years)			10

Create and solve a new Model

Trace the Efficient Frontier ☐

	Maize	Wheat	Lucerne	Cattle
1	0.0%	0.0%	0.0%	0.7%
2	0.0%	0.0%	0.0%	0.0%
3	0.0%	0.0%	17.6%	-0.4%
4	54.7%	0.0%	0.0%	1.2%
5	0.0%	0.0%	0.0%	-4.2%
6	0.0%	0.0%	8.8%	-9.4%
7	0.0%	0.0%	0.0%	-2.7%
8	0.0%	0.0%	0.0%	2.5%
9	0.0%	0.0%	-23.5%	-2.5%
10	0.0%	4.3%	0.0%	6.1%
11	0.0%	0.0%	0.0%	8.9%
12	0.0%	0.0%	-3.5%	-3.7%
13	0.0%	0.0%	0.0%	10.3%
14	0.0%	0.0%	0.0%	0.9%
15	0.0%	0.0%	-1.8%	-2.7%
16	2.6%	0.0%	0.0%	-6.8%
17	0.0%	0.0%	0.0%	-5.8%
18	0.0%	0.0%	4.2%	-0.9%
19	0.0%	0.0%	0.0%	16.7%
20	0.0%	0.0%	0.0%	-6.1%
21	0.0%	0.0%	-15.7%	5.1%
22	0.0%	15.5%	0.0%	1.9%

Appendix 2 – An extract of monthly agricultural product prices

Month	Maize (R/ton)	Wheat (R/ton)	Lucerne (R/ton)	Cattle (c/ton)	Sheep (c/ton)	Pigs (c/ton)	Barley (R/ton)
Jan-95	387.02	754.90	388.33	802.60	1235.70	617.80	671.79
Feb-95	387.02	754.90	388.33	808.10	1236.30	613.70	671.79
Mar-95	387.02	754.90	388.33	801.20	1119.00	528.80	671.79
Apr-95	387.02	754.90	456.67	797.60	1056.10	487.70	671.79
May-95	598.62	754.90	456.67	807.00	1011.00	490.80	671.79
Jun-95	598.62	754.90	456.67	773.50	990.00	481.50	671.79
Jul-95	598.62	754.90	496.67	701.00	974.20	466.90	671.79
Aug-95	598.62	754.90	496.67	682.30	1041.50	481.80	671.79
Sep-95	598.62	754.90	496.67	699.60	999.10	510.40	671.79
Oct-95	598.62	754.90	380.00	682.40	1049.20	578.20	671.79
Nov-95	598.62	787.58	380.00	723.80	1058.80	562.90	720.11
Dec-95	598.62	787.58	380.00	788.40	1205.00	627.40	720.11

Source: The South African Department of Agriculture

Appendix 3 – Extract (shown in three parts) of the output screen contained in the sheet called "Model"

Products	Cat	Optimal Weights	Current Weights	Expected Return	Average Return	Standard Deviation	Gross Margin	Adjusted Return	Optimal Area [Ha]	Current Area [Ha]	Variance Area [Ha]
Maize	A	(0.00%)		7.70%	0.64%	7.60%	5.00%	0.39%	(0)		(0)
Wheat	A	(0.00%)	5.32%	5.60%	0.39%	3.15%	5.00%	0.28%	(0)	1,331	(1,331)
Lucerne	A			6.60%	0.55%	5.94%	5.00%	0.33%			
Cattle		(0.00%)	23.16%	0.70%	0.15%	4.22%	13.00%	0.09%	(0)	5,790	(5,790)
Sheep		90.00%	12.33%	2.90%	0.32%	5.49%	13.00%	0.38%	22,501	3,083	19,418
Pigs				2.00%	0.91%	7.10%	13.00%	0.26%			
Barley	A	(0.00%)	7.97%	3.30%	0.19%	1.66%	5.00%	0.17%	(0)	1,993	(1,993)
Sugar	A	8.00%	29.03%	4.30%	0.27%	1.64%	34.00%	1.46%	2,001	7,257	(5,256)
Apples	B	(0.00%)	0.58%	6.20%	0.60%	9.72%	1.00%	0.06%	(0)	145	(145)
Bananas	C	(0.00%)	1.60%	4.80%	0.49%	18.49%	23.00%	1.10%	(0)	400	(400)
Pears	B	(0.00%)	0.08%	15.60%	1.80%	14.70%	1.00%	0.16%	(0)	21	(21)
Avocados	C	2.00%	0.05%	10.40%	1.03%	21.35%	23.00%	2.39%	500	13	487
Citrus	B		1.36%	6.70%	0.56%	14.28%	1.00%	0.07%		340	(340)

Model Results	Optimal Case	Current Case		Para-meters	Change on Current
Portfolio Adjusted Return	0.50%	0.54%	≥	1.00%	6.69%
Portfolio Risk	17.25%	5.05%			241.66%
Irrigated Crops	2,000	10,581	≤	2,000	81.10%
Drip Irrigated Orchards	(0)	506	≤	1,000	100.10%
Dry Land Plantations	500	413	=	500	21.07%
Balance	22,501	13,500		21,500	66.67%
Total Productive Land	**25,000**	**25,000**		**25,000**	
Value					
Term	10				
FCFF	1.051575	1.05537			0.36%
Risk	4.910742	1.63652			200.07%
Proxy Value	0.214138	0.64489			66.79%

	Maize	Wheat	Lucerne	Cattle	Sheep	Pigs	Barley
Maize	0.00569	-0.00002	-0.00007	-0.00002	-0.00056	-0.00040	-0.00001
Wheat	-0.00002	0.00098	-0.00002	0.00007	0.00013	0.00034	0.00028
Lucerne	-0.00007	-0.00002	0.00348	0.00014	-0.00026	-0.00111	-0.00001
Cattle	-0.00002	0.00007	0.00014	0.00176	0.00027	0.00084	0.00005
Sheep	-0.00056	0.00013	-0.00026	0.00027	0.00297	0.00112	-0.00002
Pigs	-0.00040	0.00034	-0.00111	0.00084	0.00112	0.00496	0.00003
Barley	-0.00001	0.00028	-0.00001	0.00005	-0.00002	0.00003	0.00027
Sugar	-0.00032	-0.00001	-0.00000	-0.00008	0.00016	0.00008	-0.00001
Apples	0.00116	0.00017	-0.00077	-0.00003	0.00115	0.00282	0.00007
Bananas	-0.00233	-0.00022	0.00079	0.00162	0.00074	0.00181	-0.00023
Pears	0.00136	0.00033	-0.00092	-0.00016	0.00188	0.00459	0.00024
Avocados	-0.00110	0.00036	-0.00135	0.00119	0.00080	0.00519	0.00032
Citrus	-0.00236	0.00035	-0.00199	0.00056	-0.00086	0.00294	0.00009

Appendix 4 – Details of proxy value calculation

$$\text{Proxy Value} = \frac{\text{Cash Flow Factor}}{\text{Risk Factor}}$$

$$\text{Proxy Value} = \frac{(1+R_p)^t}{(1+\sigma_p)^t}$$

where:

R_p = Adjusted Portfolio Return
σ_p = Portfolio Risk
t = Cycle time

Appendix 5 – Detailed results for the base case

		Point A	Point B	Point C	Point D	Point E
Maize	A		1,008	1,370	1,190	1,609
Wheat	A	1,331			0	879
Lucerne	B		1,660	48	802	-
Cattle		5,790	3,512	3,298	3,435	2,942
Sheep		3,083	1,814	2,075	1,938	2,431
Pigs			47		0	-
Barley	A	1,993	3,992	3,630	3,810	2,512
Sugar	B	7,257	7,340	8,952	8,198	9,000
Apples	C	145	203	74	156	-
Bananas	C	400	93	308	206	428
Pears	C	21	57		22	-
Avocados	C	13			1	407
Citrus	C	340	647	618	615	166
Irrigated Crops	A	3,324	5,000	5,000	5,000	5,000
Drip Irrigated Orchards	B	7,257	9,000	9,000	9,000	9,000
Dry Land Plantations	C	919	1,000	1,000	1,000	1,000
Balance		8,873	5,373	5,373	5,373	5,373
		20,373	**20,373**	**20,373**	**20,373**	**20,373**

Appendix 6 - Shift of the Efficient Frontier

Model Results	**Current Case**	**Base Case**	**Expanded Case**	**Base on Current**	**Expanded on Current**	**Expanded on Base**
Portfolio Adjusted Return	0.66%	0.77%	0.78%	15.75%	17.28%	1.32%
Portfolio Risk	6.20%	4.35%	3.87%	-29.85%	-37.49%	-10.88%
Irrigated Crops	10,581	15,000	20,000	41.76%	89.02%	33.33%
Drip Irrigated Orchards	506	1,000	1,000	97.63%	97.63%	0.00%
Dry Land Plantations	413	1,000	1,000	142.13%	142.13%	0.00%
Balance	8,873	3,373	3,373	-61.99%	-61.99%	0.00%
Total Productive Land	**20,373**	**20,373**	**25,373**	0.00%	24.54%	24.54%
Proxy Value						
Term		10	10			0.00%
Cash Flow Factor	1.068318	1.079452	1.0805378	1.04%	1.14%	0.10%
Risk Factor	1.824194	1.530255	1.4622874	-16.11%	-19.84%	-4.44%
Proxy Value	0.585638	0.705406	0.7389366	20.45%	26.18%	4.75%
Product Area						
Maize		1,423	1,726			
Wheat	1,331					21.30%
Lucerne		829	1,003	-100.00%	-100.00%	
Cattle	5,790	1,978	1,954			20.90%
Sheep	3,083	1,395	1,419	-65.84%	-66.25%	-1.20%
Pigs				-54.75%	-53.98%	1.70%
Barley	1,993	4,629	6,678			
Sugar	7,257	8,119	10,593	132.25%	235.07%	44.27%
Apples	145	210	168	11.88%	45.98%	30.48%
Bananas	400	770	799	44.99%	15.70%	-20.20%
Pears	21	90	38	92.46%	99.77%	3.80%
Avocados	13	230	201	330.42%	81.89%	-57.74%
Citrus	340	699	794	1670.39%	1445.48%	-12.70%
				105.70%	133.54%	13.53%