APPLYING AN EXTENDED TREE KNAPSACK APPROACH TO AN OIL PIPELINE DESIGN PROBLEM

EXTENDED ABSTRACT

1. Introduction

There are many practical decision problems that fall into the category of network flow problems and numerous examples of applications can be found in areas such as telecommunications, logistics, distributions, engineering, computer science and so on. An important aspect relevant to the discussion of network models is network design. Many network models and examples usually describe problems related to existing networks. For example, determine the shortest path, or maximum flow through an existing network. Network design problems usually involve decisions regarding network topology and capacity planning to satisfy certain demands or requirements. Some of the major issues in network design include justification of a network, scope, manageability, architecture, topology, sizing, routing etc. [2]. Apart from these issues, another challenge that researchers have to deal with is to constantly try and enhance the models or to try and improve the time taken to solve these types of models. Recent studies by Van der Merwe and Hattingh [5], [6] applied an extended tree knapsack approach to local area telecommunication networks in order to try and address these challenges.

In this paper, the feasibility of representing a network design model in a tree network model and subsequently solving it using an extended tree knapsack approach is investigated. To compare and validate the proposed technique, a specific case study (an oil pipeline design problem) was chosen from the literature that can be used as a basis for the research project [1].

The remainder of this extended abstract is organized as follows. In section 2 a brief overview of the oil pipeline design problem is given. This is followed by the model
development for the design problem in section 3. Section 4 briefly presents the results and section 5 concludes the abstract.

2. The oil pipeline design problem
To investigate the feasibility of solving a network design problem using an extended tree knapsack approach, a specific case in the literature was chosen that could be used to test the proposed approach. The selected case describes an optimal oil pipeline design for the South Gabon oil field in Africa [1]. The project considers a set of offshore platforms and onshore wells, each producing a known or estimated amount of oil that needs to be connected to a port. These connections may take place directly between platforms, well sites and the port, or may go through connection points at given locations. The objective of the pipeline system is to try and reduce the cost of transporting oil to a specific port in order to allow for expansion of production to enable increased profitability – this implies that the configuration of the network and sizes of pipes must be chosen to minimize construction cost. The South Gabon oil field network consists of 33 nodes (distances between the nodes are known). These represent the offshore platforms, onshore wells, seven connection points and one port called Gamba. There are 129 potential arcs and all the oil production in this region is transported to Gamba, from where it is then exported by sea.

3. Model development for the oil pipeline design problem
An ordinary tree knapsack problem can be regarded as choosing a sub tree of a tree. A complete description and formulation can be found in [5]. The extended tree knapsack model is a more general form of the tree knapsack model. In the extended tree knapsack model there is also a cost involved in transmitting \( y_i \) units from node \( i \) to predecessor \( p_i \), say \( f_i(y_i) \) where \( f_i \) is an arbitrary function that satisfies the condition that \( f_i(0) = 0 \). This model is discussed in detail in [3] and [4].

The methodology followed in this study comprises two main steps. In the first, the network representation of the pipeline design problem was converted into a tree structure to facilitate the use of a tree knapsack method as a solution. Second, a mathematical programming model based on an extended tree knapsack model was formulated and
solved in order to be able to express an opinion on the feasibility of the proposed methodology. The following paragraphs describe the two steps.

**Converting the pipeline network into a tree network structure**

Prior to model development, the South Gabon oil field network had to be converted into a tree structure. This process involves a series of steps i.e. identification of the root node, creating adjacent node lists for each node in the network, and finally building a tree network structure by creating paths based on the adjacency lists.

The root node was given as the Port of Gamba. Next, adjacent node lists were created; an adjacent node list for a specific node is a set of nodes that are directly connected to that specific node. Following this, a tree network can be build. This is done in a breadth first manner adding child nodes level by level to the tree.

**Model development**

The next step in the proposed methodology is to formulate a mathematical programming model which is based on an extended tree knapsack model, and which will be used to solve the tree structure constructed above.

The objective function of the extended tree knapsack model was based on a fixed charge cost model and was formulated as follows

Minimize \( \sum_{(i,j)} E_{ij} \delta_{ij} + \sum_{(i,j)} a_{ij} f_{ij} \)

subject to

\( x_j - x_{pj} \leq 0, \quad j = 1,2, \ldots, n - 1, \) (contiguity)

\( \sum_{i \in S_j} x_i = 1, \quad j = 1,2, \ldots, n, \quad s_j = \{i | N(i) = j\}, \) (no duplicate nodes)

\( Dx - B(f_1 + f_2) = 0, \) (flow balance)

\( f_{ij_1} + f_{ij_2} \leq \delta_{ij} C, \quad \text{for all} \ (i,j), \) (capacity of tree)

\( 0 \leq f_{ij_1} \leq P_{ij}, \) (pipe capacity)
\[ \delta_{ij} = \begin{cases} 
1 & \text{if arc } i, j \text{ is selected,} \\
0 & \text{otherwise,} 
\end{cases} \quad \text{(arc selection)} 
\]

\[ x_i = \begin{cases} 
1 & \text{if node } i \text{ is chosen,} \\
0 & \text{otherwise,} 
\end{cases} \quad i = 1, 2, \ldots, n. \quad \text{(node selection)} 
\]

where

\( E_{ij} \) is the fixed cost associated with each arc \((i,j)\),

\( a_{ij} \) is the cost incurred of a unit of the flow \( f_{ij} \),

\( D \) is a diagonal matrix with diagonals \( d_j \) giving the production at node \( j \).

\( B \) is a node-arc incidence matrix,

\( C \) is the total capacity at the root of the tree network,

\( P_{ij} \) is the existing pipe capacity on arc \((i,j)\),

\( f_{ij} \) represents the flow between node \( i \) and its predecessor node \( j \). This flow comprises of two parts such that \( f_{ij} = f_{ij_1} + f_{ij_2} \) where \( f_{ij_1} \) represents the flow which is less than \( \varepsilon_{ij} \) and \( f_{ij_2} \) represents the flow greater than \( \varepsilon_{ij} \) (a small positive number). In the vectors \( f_1 \) and \( f_2 \), \( f_{ij_1} \) represents the flow below pipe capacity and \( f_{ij_2} \) represents the flow above existing pipe capacity. If no pipeline exists between \( i \) and \( j \) both these flows must be zero. In the case of a network expansion problem one may visualize \( f_{ij_1} \) as a flow that is available at zero cost (up to the capacity). If no pipeline exists, this capacity is zero.

4. Results

A program was written in C++ to perform the tree generation. This resulted in an indexed tree consisting of 7030 nodes and 4183 paths. The tree was then finally solved by the extended tree knapsack model using CPLEX. A solution was obtained after 70.15 seconds.

The solution obtained showed that in 6 instances the extended tree knapsack model has chosen different arcs from those in the original problem in the literature. The difference in the two solutions can be attributed to the relaxation used for the objective function that was used in the extended tree knapsack model.
To compare this study’s results meaningfully with those of Brimberg et al., [1], the cost for both solutions, was calculated using the given pipe costs. This resulted in a cost of 1423 units for the Brimberg et al., [1] study and 1461 units for the extended tree knapsack model – a deviation of 2.6%.

The results of the study can be summarized as follows.

− The extended tree knapsack model produced a result that was within 2.6% of that in the Brimberg et al., [1] study. The low percentage deviation proves that it is definitely feasible to use an extended tree knapsack approach to solve certain network design problems. By refining the cost function used in the tree knapsack model the 2.6% gap could be further reduced - an approximation was used in this paper to represent the objective function. The approximation can be made more precise at the expense of introducing more discrete variables but this was not pursued further here.

− The feasibility is further proved by the relatively short time it took to solve the model (70.15 seconds using CPLEX).

− The oil pipeline design problem was a fairly large network comprising 33 nodes and 129 arcs. With modern software, e.g. CPLEX, large network design problems may be solved in a reasonable time using the extended tree knapsack approach.

5. Conclusion
This paper considered a network design problem and investigated the feasibility of representing a network flow model as a tree network model that can be solved by an extended tree knapsack approach.

A specific case study (an oil pipeline design problem) from the literature was selected in order to test the proposed solution method. The original oil pipeline design problem was transformed into a tree structure and the tree was then solved using an extended tree knapsack model.
The results show a small deviation from the solution presented in the literature which can be attributed to an approximation used to represent the objective function in the extended tree knapsack model. A solution was obtained after a relatively short time for a fairly large network. Based on these results it was concluded that the extended tree knapsack model proved to be a feasible alternative to solve certain network design models.

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References