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A MODEL FOR SCHEDULING PROJECTS UNDER THE CONDITION OF INFLATION AND UNDER PENALTY AND REWARD ARRANGEMENTS

JOEL K. JOLAYEMI
Department of Business Administration, College of Business
Tennessee State University,
330, 10th Avenue North, Nashville, TN 37203, U.S.A.

ABSTRACT
A zero-one mixed integer linear programming model is developed for the scheduling of projects under the condition of inflation and under penalty and reward arrangements. The effects of inflation on time-cost trade-off curves are illustrated and a modified approach to time-cost trade-off analysis presented. Numerical examples are given to illustrate the model and its properties. The examples show that misleading schedules and inaccurate project-cost estimates will be produced if the inflation factor is neglected in an environment of high inflation. They also show that award of penalty or bonus is a catalyst for early completion of a project, just as it can be expected.

Keywords: Cost curves, time-cost trade-off analysis, mixed-integer linear programming, linear approximation, due date

1. INTRODUCTION

The development of the Kelly-Walker's Time-Cost Trade-off Model (TCTM) in the early fifties (see Kelly and Walker [15] and Kelly [14]) marked the beginning of the application of formal optimization techniques in project planning and scheduling. Ever since, the TCTM has gone through a lot of modifications, extensions, reviews and advancements (witness Davis [3], Dean [4], Elmaghraby [7,9], Moder [18], Russel [19], Jolayemi [12] and Jolayemi et al. [13], for a few examples).

The TCTMs are usually in the form of linear programming (LP) models. This makes them easier to solve and, therefore, more easily adaptable than other types of project-scheduling models - mainly resource-constrained models (RCM) - which are formulated in terms of integer or dynamic programming or forms of bounded and implicit enumeration (see Erenguc et al. [10], Sprecher et al. [20], Bottcher et al. [1], Gutierrez et al. [11], Brucker et al. [2] Demeulemeester et al. [5,6], Kolish [16,17], Vercellis [21] and Elmaghraby [8]). The ready availability of powerful computers and efficient operations research (OR) codes almost everywhere makes the TCTM much easy to solve when problem sizes are large.

Another inadequacy of the existing models - excluding Jolayemi et al. [13] and Jolayemi [12] - is that they do not consider the inflation factor as an important input, either explicitly or implicitly. There is hardly any country in the world with an inflation-free economy. The situation is worse in most developing countries where inflation is constantly on the rise. Neglecting the inflation factor in an environment of high inflation results in underestimation of project costs and, consequently, project failure.

The new model in this research will have the inflation factor as one of its important inputs. We shall demonstrate the effect of inflation on time-cost trade-off curves (TCTCs) and on time-cost trade-off analysis (TCTA). A modified approach to TCTA is introduced. Furthermore, numerical examples will be given to illustrate the model and its properties.

2. EFFECTS OF INFLATION ON THE TCTA, AND REDEFINITION OF PROJECT COSTS

Before we discuss the effects of inflation on the TCTA and the TCTCs, we need to briefly review the Kelly-Walker's TCTCs.

2.1 The Kelly-Walker's TCTCs

The Kelly-Walker's TCTC's (Kelly and Walker [15]) are graphs of the project-activity costs against project-activity durations or times. Conventionally, project activity costs are classsified into two categories, namely: the direct and the indirect costs. The direct costs consist of the costs of materials, equipment, and direct labour while the indirect costs consist of the cost of supervision, capital, inventory, insurance, penalty for late project completion and bonuses for early completion. The direct costs decrease with increase in activity duration while the indirect costs increase.

The direct cost decreases with time because the more the time available for the execution of a project, the less the costs of overtime, workshifts, and extra labour. Costs of materials and equipment also decrease because good availability of time allows ordering, procurement, and shipping of materials and equipment to be done through the cheapest but not necessarily the fastest means.

However, when project or activity duration increases, more costs are incurred on capital (tied-down capital) inventory-insurance, pilferage, deterioration, and project supervision.

Figures 1 and 2 shows the cost-graphs and the TCTC.

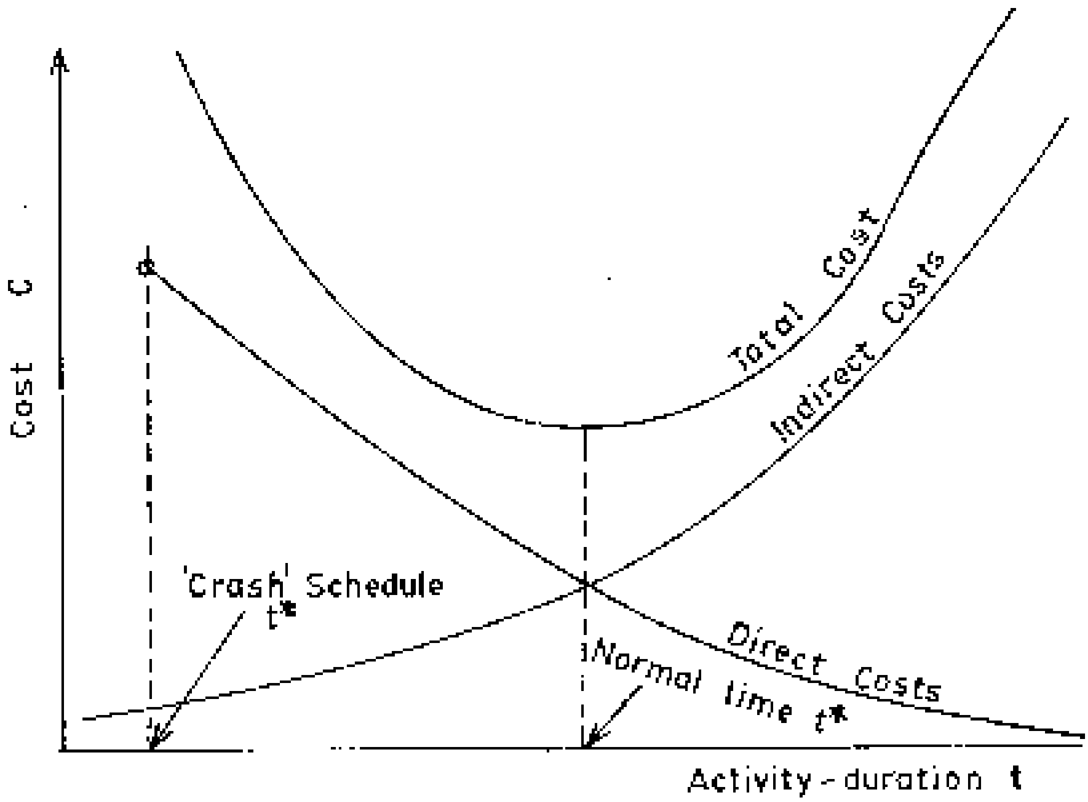

Figure 1: Graphs of the direct, indirect and total costs

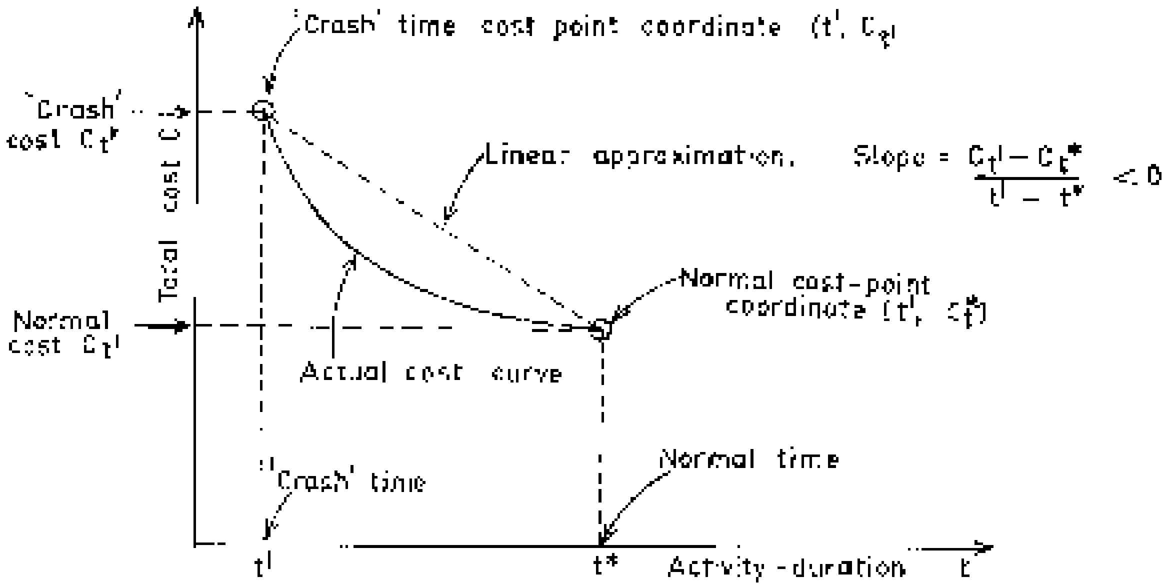

Figure 2: The time-cost trade-off curve

The normal cost is the minimum point on the total cost-curve. It is the minimum total cost of the direct and the indirect costs of an activity. The corresponding activity duration t* is the normal time. The 'crash' time is the time that is technically possible for executing an activity. The 'crash' cost is the total cost required to achieve the crash performance time. The total-cost curve is the cost-function to minimize.

To make the application of LP possible, the total cost curve is approximated by a straight line that passes through the `crash' point and the normal point. The equation of the linear approximation is easily determined after determining its slope. This equation constitutes the objective function of the LP problem.

2.2 The Effects of Inflation on the TCTCs and TCTA

When there is high inflation, the direct costs - particularly costs of equipment and materials - increase with time, instead of decreasing. The indirect costs increase with time, as before. Consequently, the total activity costs increase rapidly with time. The TCTCs that explain this situation is shown below in Figure 3.

The direct and the indirect costs may not cross if inflation is very high. When they show upward trends, as shown in Figure 3, the total cost curve will not be convex and hence there will be no normal point. However, the `crash' point can always be determined. Due to the non-existence of the normal time, the linear approximation to the TCTC and the cost slope cannot be uniquely determined. Therefore, the TCTA cannot produce any reliable result and, hence, the time-cost trade-off model is not applicable unless the existing costs are redefined.

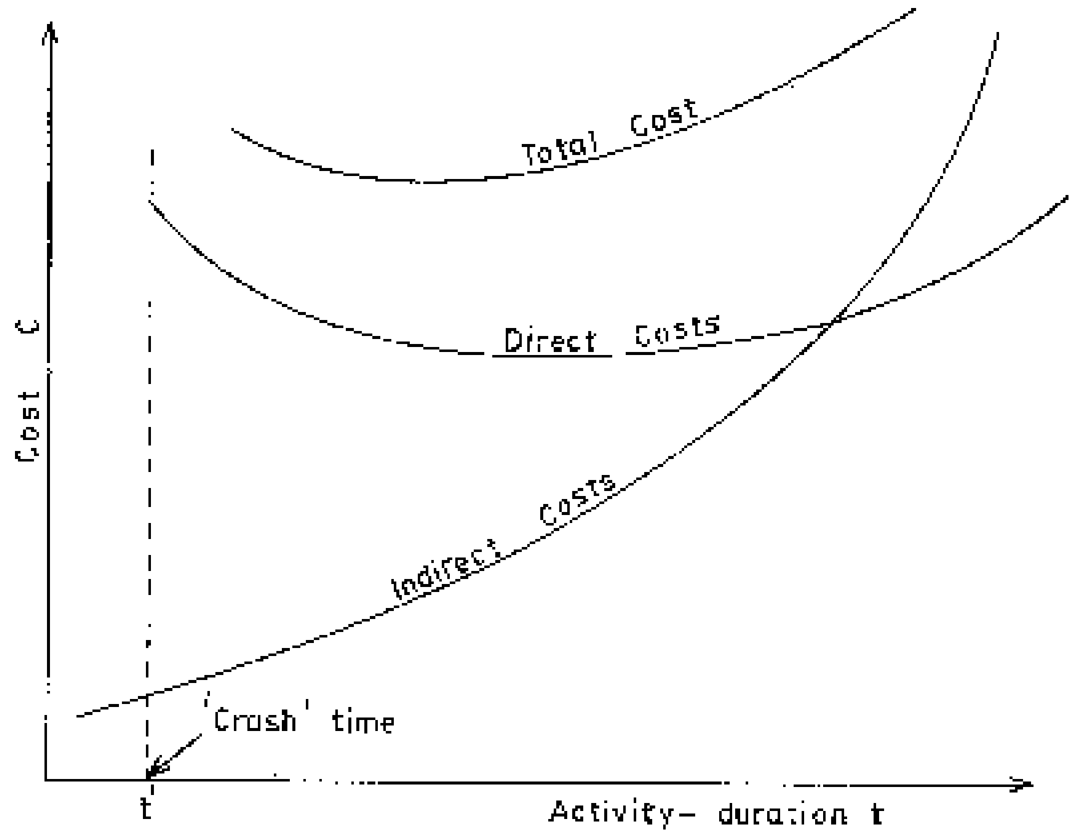

Figure 3: Graphs of the direct, the indirect and the total cost curves under the conditions of inflation

2.3 Redefinition of the activity costs

The redefinition of the activity costs involves breaking the direct cost into two parts (see Jolayemi et al [13]). The first part, denoted the direct cost A, is made up of costs of equipment and materials. Costs of labour constitute the second part and is denoted the direct cost B. The direct cost A increases with rising inflation. The effect of inflation on the direct cost B is not readily apparent. In fact, the direct cost B may decrease with time - since the more the time available for the execution of a project, the less the costs of overtime, workshifts, and extra labour (hired temporary labour).

To make TCTA applicable, we have excluded the direct cost A from the graphical analysis of costs. The graphs of the direct cost B and the indirect cost is as shown in Figure 4.

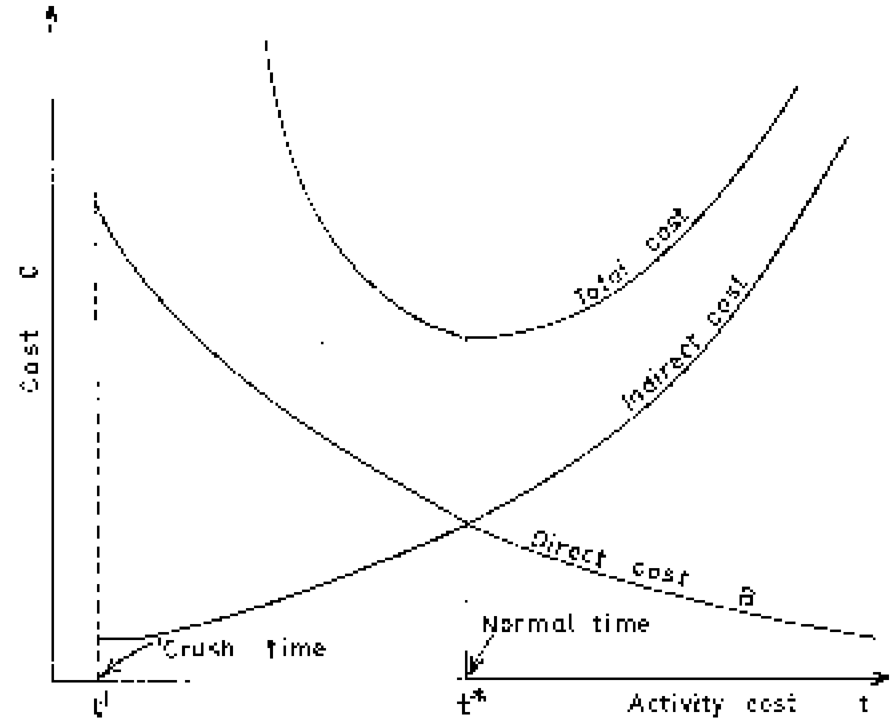

Figure 4: Graph of the direct cost B and the indirect cost

Figure 4 now has the form of the Kelly-Walker's TCTCs. Therefore, it can be used for the TCTA. The linear approximation to the total cost curve, its gradient, and equation are derived from Figure 5.

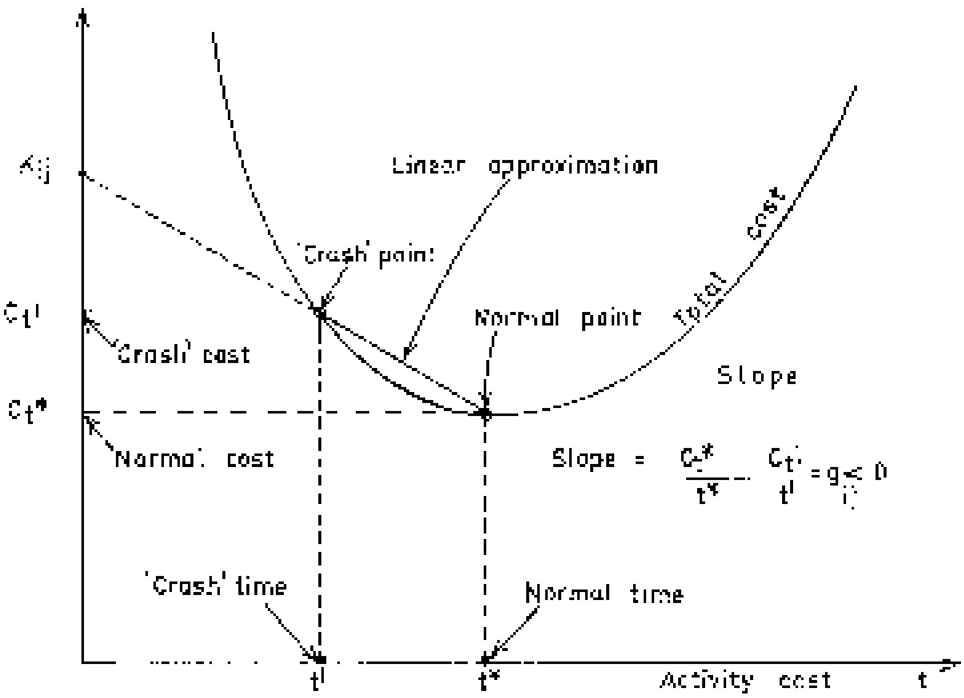

Figure 5: Linear approximation to the total cost curve.

3. MODEL FORMULATION

We first define the decision variables and parameters of the model before presenting the mathematical model. Our definitions and model formulation are based on activity-on-arcs project network.

3.1 Decision Variables

x_{ij} : The duration (in years) for activity (i, j).

T_i : The earliest start-time (in years) for activity (i, j).

T_j : The completion time for activity (i, j).

T_1 : The earliest start-time for the project.

T_n : The total project completion time in years.

z_t : A binary variable, which is 1 if the project is completed in time t and 0 otherwise.

3.2 Indices and Parameters

d : The project due date. It is the time officially agreed upon by the project owner and his contractor for the completion of the project.

E : The earliest time that is technically possible for the completion of the project. Thus, $E \leq d$.

λ : The constant constraint placed on the total project duration. It is the maximum allowable time for the completion of the project.

t : The instantaneous point-in-time that the project is completed. t is one of the partition points of the time interval from E to λ. The discretisation of t will make it possible for us to formulate the integer component of our model. To ensure that this discretisation does not affect the accuracy of the solution to the model, the constant difference between successive values of t has been defined to be small.

ϵ : The constant difference between successive values of t.

B_t : The reward or bonus in dollars for completing the project in period t before the expiration of the due date d. We assume that B_t decreases as t increases from E to d with step-size ϵ.

P_t : The penalty in dollars for completing the project in period t after the expiration of the due date d. It is assumed that P_t increases as t increases from $d+\epsilon$ to λ with step-size ϵ.

(i,j) : An activity that starts at node i and terminates at node j of the project network, i less than j.

g_{ij} : The cost slope or gradient of the linear approximation to the portion of the total cost curve that lies between the normal and the 'crash' points with respect to activity (i,j).

R_i : The set of all items and equipment needed for the execution of activity (i,j). If y numbers of an item r are needed in node i, each of the y items are included in the set R_i as a separate entity.

V : The set of all nodes of the project, i.e., $V = \{i\}_{i=1}^{n}$:

p_{ir} : The price of item r (equipment or materials) at node i of the project network. $p_{ir} = 0$ if $r \notin R_i$. It is assumed that an item r needed for the execution of activity (i,j) is procured at node i just before the commencement of the activity.

A : The set of all activities (i,j) of the project, $i,j \in V$

k_{ij} : The intercept made with the vertical axis by the linear approximation to the portion of the total cost curve that lies between the normal and the 'crash' points with respect to activity (i,j).

L_{ij} : The normal time for activity (i,j).

l_{ij} : The 'crash' time for activity (i,j).

n : The last node of the project's network.

θ : Annual inflation rate. It can be continuous or discrete, but we shall assume a continuous inflation rate.

3.3 The Model

The linear approximation to the total cost curve for an activity (i,j), ($(i,j) \in A$) is $k_{ij} + g_{ij}x_{ij}$. Therefore, the sum of the costs of labour and indirect costs for the project is

$$\sum_{(ij)\in A} (k_{ij} + g_{ij}x_{ij})$$

Since the inflation rate is θ, the cost of item r required to be bought at node i (at the beginning of activity (i,j)) time T_i from now is $p_{ir}e^{\theta T_i}$. The total cost of all items needed for the execution of the whole project is

$$\sum_{i\in V} \sum_{r\in R_i} p_{ir}e^{\theta T_i}$$

The first term of Taylor's expansion of this non-linear term (see Russel [19] Jolayemi[12], and Jolayemi et al [13]) is

$$\sum_{i\in V} \sum_{r\in R_i} p_{ir}(1 + \theta T_i)$$

Therefore, the linearized total cost of executing the project is

$$\sum_{(ij)\in A}(k_{ij}+g_{ij}x_{ij})+\sum_{i\in V}\sum_{r\in R_i}p_{ir}(1+\theta T_i)$$

Let the time-interval from E to λ be partitioned into m equal smaller intervals. Let G be the set of the partition-points (the time-points including points E and λ) $E+\epsilon, E+2\epsilon, ..., E+(m-1)\epsilon, \lambda$, where ϵ is as defined earlier.

Define:

$$Q=\{t : E\leq t\leq d | t\in G\}$$

and

$$W=\{t : d<t\leq \lambda | t\in G\}$$

The bonus awarded the contractor for completing the project before the expiration of the due date d is

$$\sum_{t\in Q}B_t z_t$$

The penalty awarded against the contractor for not completing the project before the expiration of the due date is

$$\sum_{t\in W}P_t z_t$$

Therefore, the total cost to minimize (the objective function) is

$$\sum_{(i,j)\in A}(k_{ij}+g_{ij}x_{ij})+\sum_{i\in V}\sum_{r\in R_i}p_{ir}(1+\theta T_i)-\sum_{t\in Q}B_t z_t+\sum_{t\in W}P_t z_t$$

For each activity (i,j), the difference between the earliest event time T_i and the latest event time T_j must be, at least, as large as x_{ij}. Hence, the first constraint is

$$T_i+x_{ij}-T_j\leq 0 \quad \text{for all} \quad (i,j)$$

The normal time L_{ij} must be greater than or equal to the scheduled activity duration x_{ij}. Therefore, the second constraint is

$$x_{ij}\leq L_{ij} \quad \text{for all} \quad (i,j)$$

The scheduled activity duration must be greater than or equal to the 'crash'-time l_{ij}. Therefore, the third constraint is

$$-x_{ij}\leq -l_{ij} \quad \text{for all} \quad (i,j)$$

The time interval between the earliest time T_1 that the project begins and the completion time T_n must be less than or equal to λ. Therefore, the fourth constraint is

$$T_n-T_1\leq \lambda$$

The completion of the project should result in the award of a penalty or a bonus (but not both). Therefore, the sixth constraint is

$$\frac{-\epsilon}{2} \leq \sum_{t \in G} t z_t - T_n \leq \frac{\epsilon}{2}$$

Note that $\frac{1}{2}\epsilon$ has been used here instead of ϵ to ensure that the neighbourhood of t covers only the mid-points between t and $t-1$, and t and $t+1$ respectively.

Putting everything together, the resulting mixed integer linear programming model (MILP) is:

$$\text{Minimize } z = \sum_{ij \in A} (k_{ij} + g_{ij} x_{ij}) + \sum_{i \in V} \sum_{r \in R_i} p_{ir}(1 + \theta T_i) - \sum_{t \in Q} B_t z_t + \sum_{t \in W} P_t z_t \qquad \mathbf{(3-1)}$$

Subject to:

$$\begin{aligned}
T_i + x_{ij} - T_j &\leq 0 \quad \text{for all} \quad (i,j) \in A && \mathbf{(3-2)} \\
x_{ij} &\leq L_{ij} \quad \text{for all} \quad (i,j) \in A && \mathbf{(3-3)} \\
-x_{ij} &\leq -l_{ij} \quad \text{for all} \quad (i,j) \in A && \mathbf{(3-4)} \\
T_n - T_1 &\leq \lambda && \mathbf{(3-5)} \\
\sum_{t \in G} z_t &= 1 && \mathbf{(3-6)} \\
-\frac{\epsilon}{2} &\leq \sum_{t \in G} t z_t - T_n \leq \frac{\epsilon}{2} && \mathbf{(3-7)}
\end{aligned}$$

$x_{ij}, T_i, T_j, T_n \geq 0$ for all $(i,j) \in A$ and all $i, j \in V$, $z_t = 0$ or 1 for all $t \in G$ and $p_{nr} = 0$ for all $r \in R_i$.

To reduce the constraints of the MILP, we use the transformation $y_{ij} = x_{ij} - l_{ij}$ (see Jolayemi et al [13] and Jolayemi [12] to obtain the following transformed MILP model:

$$\text{Minimize } x_o = \sum_{(i,j) \in A} (a_{ij} + g_{ij} y_{ij}) + \sum_{i \in V} \sum_{r \in R_i} p_{ir}(1 + \theta T_i) - \sum_{t \in Q} B_t z_t + \sum_{t \in W} P_t z_t \quad \mathbf{(3-8)}$$

Subject to:

$$\begin{aligned}
T_i + y_{ij} - T_j &\leq -l_{ij} \quad \text{for all} \quad (i,j) \in A && \mathbf{(3-9)} \\
y_{ij} &\leq L_{ij} - l_{ij} \quad \text{for all} \quad (i,j) \in A && \mathbf{(3-10)} \\
T_n - T_1 &\leq \lambda && \mathbf{(3-5)} \\
\sum_{t \in G} z_t &= 1 && \mathbf{(3-6)} \\
-\frac{\epsilon}{2} &\leq \sum_{t \in G} t z_t - T_n \leq \frac{\epsilon}{2} && \mathbf{(3-7)}
\end{aligned}$$

$y_{ij}, T_i, T_j, T_n \geq 0$ for all $(i, j) \in A$ and all $i, j \in V$ and $T_1 = 0$.

$z_t = 0$ or 1 for all $t \in G$.

$a_{ij} = k_{ij} + g_{ij} l_{ij}$ and $g_{ij} \leq 0$.

4. NUMERICAL EXAMPLES

4.1 Problem

Table 1 in appendix A shows the list of four condensed activities of a hypothetical research and development project and the input parameters of the corresponding LP problem. The materials needed at the beginning nodes of each activity and their costs are given in table 2 of appendix A. Time estimates are in years. Table 3 in the appendix presents data on penalties and bonuses.

4.2 Solution

The transformed model for this numerical example is presented in appendix B. The LINDO software program was used to obtain the optimal solution to the MILP problem. The optimal values of the decision variables are shown on the arcs of the network of Fig. 6 and in Table 1 below

The optimal event times are shown on the nodes. The numbers in braces are the "crash" and the normal times. The values of the original variables, the x_{ij}'s, have been obtained by reversing the original transformation. To minimize the total project cost, the activity durations on the arcs where procurements of materials and equipment are made take the values of their respective "crash" time. On the other hand, the activity durations on the arcs that enter the node (node 5) where no procurement is made take the values of their respective activity normal times. With this optimal schedule, the optimal project completion time T_5 is 1.06.

If there had been no inflation, the optimal value of T_5 would have been 2.0 or as close to 2.0 as possible for the objective function to be a minimum. This result shows that when there is inflation, a contractor must complete his project very early (earlier than the due date) to reduce the project cost.

The hypothetical contractor in this example is awarded a bonus of \$200 for completing the project before the due date. The total project cost is \$800.79.

Table 1: Optimal solution to the numerical example.

Decison variables	Optimal values	Decision variables (binary)	Optimal values
x_{12}	0.1	$z_{1.0}$	1
x_{23}	0.2	$z_{1.2}$	0
x_{24}	0.16	$z_{1.4}$	0
x_{35}	0.6	$z_{1.6}$	0
x_{45}	0.8	$z_{1.8}$	0
T_1	0.0	$z_{2.0}$	0
T_2	0.1	$z_{2.2}$	0
T_3	0.3	$z_{2.4}$	0
T_4	0.26	$z_{2.6}$	0
T_5	1.06	$z_{2.8}$	0
		$z_{3.0}$	0

4.3 Further Study of the Model's Structure

4.3.1 Cases

We shall consider the following cases for a more detailed study of the model's structures.

Case 1: Bonus arrangement only. No penalty.

Case 2: Penalty arrangement only. No bonus.

Case 3: No penalty, no bonus, and no inflation.

Case 4: No inflation, but there is penalty and reward.

To compare the results here with the results in section 4.2, we have used the same input data used in that section. To save space, we will not present the corresponding models for the 4 cases. The optimal solution to the LP problem under each case is shown in table 2 below.

Table 2. Optimal solutions to the numerical examples illustrating each of the cases.

Decision variables	Optimal values of the decision variables Case 1	Case 2	Case 3	Case 4
x_{12}	0.1	0.1	0.5	0.5
x_{23}	0.2	0.2	0.9	0.2
x_{24}	0.16	0.16	0.7	0.45
x_{35}	0.6	0.6	0.6	0.4
x_{45}	0.8	0.8	0.8	0.15
T_1	0.0	0.0	0.0	0.0
T_2	0.1	0.1	0.5	0.5
T_3	0.3	0.3	1.4	0.7
T_4	0.26	0.26	1.20	0.95
T_5	1.06	1.06	2.0	1.1
$z_{1.0}$	1	1	1	1
$z_{1.2}$	0	0	0	0
$z_{1.4}$	0	0	0	0
$z_{1.6}$	0	0	0	0
$z_{1.8}$	0	0	0	0
$z_{2.0}$	0	0	0	0
$z_{2.2}$	0	0	0	0
$z_{2.4}$	0	0	0	0
$z_{2.6}$	0	0	0	0
$z_{2.8}$	0	0	0	0
$z_{3.0}$	0	0	0	0
Bonus/ penalty	(bonus) \$200.00	(bonus) \$200.00	(bonus) None	(bonus) \$200.00
Objective function	\$800.79	R1000.79	\$976.8	\$770.4

(Note that where bonus is indicated, the value of the bonus is subtracted from the total project cost to give the value in the last row).

4.3.2 Comments on the results obtained for each case

Case 1:

The results here show that the optimal values of activity and project durations, and total costs, are the same with the values obtained in the example of section 4.2. The reasons for this is easy to see. Due to inflation, the durations of activities that enter the nodes where procurements are made have to take the values of their respective 'crash' time for the total cost to be minimum. Also, the value $T_n^* = 1.06$ falls within the interval where the highest value of bonus is awarded for early project completion. This value of T_n^* still allows x_{35} and x_{45} to take their normal time values of .6 and .8 respectively. The conclusion from this is that inflation and/or penalty and reward arrangements make(s) the execution of a project to be accelerated.

Case 2:

The optimal values obtained for the decision variables are the same with those obtained in case 1. This is so for the same reasons given in case 1 in relation to inflation. That the optimal project completion time $T_n^* = 1.06$ is far less than $t = 2.2$ - the time at which penalty charges against the contractor start - shows that penalty charges against project completion is not at all consequential in this case. Inflation is the major factor for the optimal schedule produced. The optimal total project cost is higher here than in case 1 (\$1000.79 instead of \$800.79) because no bonus is awarded for early project completion, unlike in case 1.

Case 3:

The optimal values differ greatly from those obtained in the two previous cases, including the example in section 4.1. The optimal activity durations are the normal time values for their corresponding activities. The optimal value of project duration T_5 is 2.0 and this is the highest value obtained for T_5 in all the examples.

The values of the objective function, including the costs of materials is \$976.8 (remember no bonus). These results show that misleading schedules and inaccurate project cost estimate will be produced if the inflation factor is neglected in an environment of high inflation.

Case 4:

The optimal project duration in this case is $T_5^* = 1.1$. This value is not different from those obtained in cases 1 and 2 and the example in section 4.1. However, the optimal values obtained for the activity durations here are different from any of those obtained in the previous cases. The optimal values for x_{35} and x_{45} are no longer their corresponding normal-time values, and the optimal values of x_{12}, x_{23} and x_{24} are no longer their respective 'crash'-time values.

The inclusion of penalty and reward arrangements has produced a shorter value of project duration here than in case 3. This shows that penalty and reward arrangements encourage early project completion in an inflation-free environment.

5. CONCLUSION

The model developed in this paper will find useful applications in construction industries. It will also be useful in manufacturing industries, particularly in the areas of research and development, plant expansion, rehabilitation and equipment maintenance.

This is the third project scheduling model in literature (see Jolayemi et al [13] and Jolayemi [12]) that has ever explicitly considered the inflation factor as an important input. It is also the only model of the time-cost trade-off type that incorporates penalty and reward arrangements. The model constitutes a major advancement on the popular Kelly-Walker's model.

The ready availability of many efficient commercial software packages like spreadsheets, GAMS, Lindo, SAS-OR, CPLEX and OSL that can be used to solve the model makes it very adaptable and applicable.

APPENDICES

APPENDIX A.

INPUT DATA FOR THE NUMERICAL EXAMPLES

Table 1: Project activities and estimates of input parameters.

Activity(i,j)	k_{ij}	g_{ij}	L_{ij}	l_{ij}
(1,2)	60	−8	0.5	0.1
(2,3)	70	−5	0.9	0.2
(2,4)	68	−7	0.7	0.16
(3,5)	40	−6	0.6	0.04
(4,5)	55	−4	0.8	0.15

Table 2: Materials needed at the beginning node i and their costs

Types of materials needed and costs in dollars.								
	1	2	3	4	5	6	7	Total
Node I	(2)	(3)	(7)	(8)	(4)	(6)	(5)	cost ($)
1	4	5	-	8	9	-	10	173
2	3	8	4	6	-	8	9	199
3	5	6	5	-	3	10	7	170
4	7	3	2	7	5	4	5	162

The value of the continuous inflation index is .12

Table 3: Completion time (partition points) and corresponding bonus/penalty awarded.

Before due date		After due date	
Completion time	Bonus	Completion time	Penalty
1.0	200	2.2	150
1.2	180	2.4	170
1.4	160	2.6	190
1.6	140	2.8	210
1.8	120	3.0	230
2.0	100		

$E = 2, d = 2.0$

APPENDIX B.

TRANSFORMED LP MODEL FOR THE NUMERICAL EXAMPLE.

$$
\begin{aligned}
\text{Minimize } z \;=\; & 993.24 - 8y_{12} - 5y_{23} - 7y_{24} - 6y_{35} \\
& + 20.4T_3 + 19.44T_4 - 200x_{1.0} - 180x_{1.2} \\
& - 160x_{1.4} - 140x_{1.6} - 120x_{1.8} - 100x_{2.0} \\
& + 150x_{2.2} + 170x_{2.4} + 190x_{2.6} \\
& + 210x_{2.8} + 230x_{3.0}
\end{aligned}
$$

subject to:

$$
\begin{aligned}
y_{12} - T_2 &\le -0.1 \\
T_2 + y_{23} - T_3 &\le -0.2 \\
T_2 + y_{24} - T_4 &\le -0.16 \\
T_3 + y_{35} - T_5 &\le -0.04 \\
T_4 + y_{45} - T_5 &\le -0.15 \\
y_{12} &\le 0.4 \\
y_{23} &\le 0.7 \\
y_{24} &\le 0.54 \\
y_{35} &\le 0.56 \\
y_{45} &\le 0.65 \\
T_5 - T_1 &\le 3.0
\end{aligned}
$$

$$
\begin{aligned}
z_{1.0} + z_{1.2} + z_{1.4} + z_{1.6} + z_{1.8} + z_{2.0} + z_{2.2} + z_{2.4} + z_{2.6} + z_{2.8} + z_{3.0} &= 1 \\
z_{1.0} + 1.2z_{1.2} + 1.4z_{1.4} + 1.6z_{1.6} + 1.8z_{1.8} + 2.0z_{2.0} + 2.2z_{2.2} + 2.4z_{2.4} & \\
+ 2.6z_{2.6} + 2.8z_{2.8} + 3.0z_{3.0} - T_5 &\le 0.1 \\
z_{1.0} + 1.2z_{1.2} + 1.4z_{1.4} + 1.6z_{1.6} + 1.8z_{1.8} + 2.0z_{2.0} + 2.2z_{2.2} + 2.4z_{2.4} & \\
+ 2.6z_{2.6} + 2.8z_{2.8} + 3.0z_{3.0} - T_5 &\ge -0.1
\end{aligned}
$$

$$
\begin{aligned}
y_{ij} &\ge 0 \quad \text{for all} \quad (i,j) \in A, T_1 = 0, z_t = 0 \quad \text{or} \quad 1 \quad \text{for all} \quad t \in G, \\
T_i &\ge 0 \quad \text{for} \quad i = 2, 3, 4, 5.
\end{aligned}
$$

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